Simulation of Landing Gear Dynamics and Brake-Gear Interaction

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Abstract

The term landing gear indicates one of the main functions of the gear, namely the containment of the landing impact but it fails to describe the other functions, such as provision of means for the aircraft to maneuver on the ground, taxi and take off. Landing gear dynamics, especially shimmy and brake-induced vibrations, is one of the problems faced today by the aircraft community. Landing gear vibrations may lead to fatal accidents due to excessive wear, it can also shorten the gear life, and affect comfort to the pilot and passengers. Among the most important reasons for landing gear vibrations are unsuitable combination of structural stiffness, damping, and pneumatic tire characteristics further more an unlucky combination of brake system design with the tire physics can produce a serious vibration problem. Shimmy may be caused by a number of conditions such as low torsional stiffness, excessive free play in the gear, wheel imbalance, or worn parts. Although equations for representing various parts of a landing gear are well established, solving the problems manually with mathematical programs can be slow and laborious. However, many commercially available computer-aided engineering tools have made it possible to test some of the problems in the design phase by simulating the landing gear impact and ground maneuvers. At the German Aerospace Center (DLR), simulation of such an unstable and complex phenomenon during aircraft ground maneuvers is done to detect vibrations in aircraft landing gear and study the effect of important parameters that may affect the stability and comfort. A multi-body simulation (MBS) model of a landing gear was prepared using Nastran for this purpose and imported into the MBS tool SIMPACK. An adequate modeling of tire and brake dynamics is an important issue for the analysis of the behavior of an aircraft during ground maneuvers as potentially unstable phenomenon such as gear walk and shimmy may occur in these phases and was also implemented into the MBS tool. With the help of all these modules different ground maneuvers were simulated including braked maneuver. Effect of various parameters on landing gear vibrations was also studied. Finer points are highlighted in the conclusion which may require more detailed study and analysis.

Zusammenfassung

Die Bezeichnung Fahrwerk kennzeichnet eine der Hauptfunktionen des Antriebs, nämlich die Beherrschung des Landungsstoßes, aber es kann nicht die anderen Funktionen, wie die Bereitstellung von Mitteln für das Flugzeug, um auf dem Boden zu manövrieren, rangieren und abzuheben beschreiben. Fahrwerksdynamik, besonders shimmy (flattern) und von den Bremen verursachte Vibrationen sind nur einige der Probleme, den sich die Flugzeughersteller heute stellen müssen. Vibrationen des Fahrwerks können zu fatalen Unfällen aufgrund übermäßiger Abnutzung, führen, sie können auch die Lebensdauer des Antriebs verringern und den Komfort des Piloten und der Passagiere beeinflussen. Unter den wichtigsten Gründen der Fahrwerksvibrationen sind ungeeignete Kombinationen von baulicher Steife, Dämpfung und pneumatischen Reifenchrakteristika, ferner kann eine unglückliche Kombination von Bremssystementwurf mit der Physik der Reifen ein erhebliches Vibrationsproblem hervorrufen. Das Flattern (Shimmy) kann durch eine Anzahl von Bedingungen, wie eine geringe Drehmomentensteifigkeit, übermäßiges Spiel im Antrieb, Reifenunwucht oder abgenutzten Teilen, hervorgerufen werden. Obwohl Gleichungen, die die verschiedene Teile eines Fahrwerks darstellen, gängig sind, kann das lösen der Probleme von Hand mit mathematischen Programmen langsam und umständlich sein. Dennoch haben es viele kommerziell verfügbare rechnergestützte Entwicklungswerkzeuge (CAD) möglich gemacht, einige der Probleme in der Entwurfsphase durch die Simulation des Landungsstoßes und Bodenmanöver zu testen. Beim Deutschen Zentrum für Luft- und Raumfahrt (DLR) wurde die Simulation solch eines instabilen und komplexen Phänomens während des Flugzeugbodenmanövers durchgeführt, um Vibrationen im Fahrwerk zu ermitteln und um die Effekte wichtiger Parameter zu studieren, die die Stabilität und den Komfort beeinflussen könnten. Ein Mehrkörpersimulations(MKS)-Model eines Fahrwerks wurde unter der Verwendung von NASTRAN für diesen Zweck erstellt und in das MKS-Werkzeug SIMPACK importiert. Eine angemessene Modellierung der Reifen- und Bremsdynamik ist eine wichtige Angelegenheit für die Analyse des Verhaltens eines Flugzeugs während der Bodenmanöver, da potenziell instabile Phänomene, wie Fahrwerkslängsschwingungen und Flattern (Shimmy) in dieser Phase vorkommen können und ebenfalls in das MKS-Werkzeug implementiert wurde. Mit Hilfe all dieser Module wurden verschiedene Bodenmanöver inklusive Bremsmanöver simuliert. Der Effekt verschiedener Parameter auf Fahrwerksvibrationen wurde ebenfalls studiert. Genauere Punkte werden im Abschluss hervorgehoben, die detailliertere Studien und Analysen erfordern können.

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Chapter 1

Introduction

1.1 Landing Gear Dynamics - Problem Definition

The term landing gear indicates one of the main functions of the gear, namely the containment of the landing impact but it fails to describe the other functions, such as the provision of means for the aircraft to maneuver on the ground, taxi and take off [12]. The predominant task of an airplane is no doubt to fly with the best performance achievable. It must not be forgotten, however, that it will spend a good part of its life on the ground. Landing gear dynamics, especially shimmy and brake-induced vibrations, is one of the problems faced today by the aircraft community. Though they are not catastrophic, can lead to fatal accidents due to excessive wear. It can also shorten the gear life and cause discomfort to the pilot and passengers. Structures of modern aircraft become increasingly flexible. The main reasons are slender fuselages that frequently arise from the stretching of existing aircraft, see [38], and the use of new, light-weight structures and materials that influence the vibrational properties of fuselage and wings. Not only unsuitable combination of structural stiffness, damping, and pneumatic tire characteristics but also an unlucky combination of brake system design with the tire physics can produce a serious vibration problem [44].

Shimmy may be caused by a number of conditions such as low torsional stiffness, excessive free play in the gear, wheel imbalance, or worn parts. Brake-induced vibration includes conditions known as gear walk, squeal and chatter which are caused by the characteristic friction between the brake rotating and non-rotating parts. These phenomena will be explained in details later in Chapter 2.

Although equations for representing various parts of a landing gear are well established, solving the problems manually with mathematical programs can be slow and laborious. Simplifications made to reduce problem size may introduce inaccuracies such that a design modification to correct a problem in one area causes unforeseen vibration in other parts of the structure. In many cases, vibration problems may not be uncovered until physical prototypes are built and tested, adding considerable time and expense to the product development cycle.

However, many commercially available Computer-Aided Engineering(CAE) tools have made it possible to test some of the problems in the design phase by simulating the landing gear impact and ground maneuvers. An adequate modeling of tire and brake dynamics is an important issue for the analysis of the behavior of an aircraft during ground maneuvers as potentially unstable phenomenon such as gear walk and shimmy may occur in these phases. At the German Aerospace Center (DLR), simulation of such an unstable and complex phenomenon during aircraft ground maneuvers is done to detect vibrations in aircraft landing gear. A commercial multi-body simulation tool SIMPACK is used for this purpose. It allows the import of external models from other codes such as Nastran. Landing gear parts modeled in Nastran are used to represent the vibration modes accurately. The goal of this work is to study brake and gear interaction and the related vibration phenomena including low frequency gear walk, and shimmy. The parameter study affecting landing gear vibrations is also done.

1.2 Earlier Approaches

Both civil and military organizations have put great effort into optimization of the landing gear and its components. There exist some specific publications in the area of landing gear dynamics and simulation. This can be divided into three broad modules, namely modeling and simulation related to *landing gear structure*, *tire*, and *brake dynamics*.

An early overview of computer simulation of aircraft and landing gear is given by Doyle [14]. Shepherd, Catt, and Cowling [10] describe a program funded by British Aerospace for the analysis of aircraft-landing gear interaction with a high level of detail, including brakes and anti-skid, steering control, to simulate standard hardware rig test (dynamo-meter and drop tests) as well as flight tests involving ground contact. Barnes and Yager [2] discuss the use of simulators for aircraft research and development. Two publications of the IAVSD (International Association for Vehicle System Dynamics), Hitch [29] in 1981 and Krüger et al [41] in 1997 and one at NASA Langley Research Center by Pritchard [53] are state-of-the-art overviews of aircraft landing gear dynamics.

Modeling tires is a science for itself: In 1941, von Schlippe and Dietrich [60], analyzed the shimmy motion of an aircraft tire and described the interaction of tire and landing gear leg stiffness with tire forces analytically. Pacjeka [51] used a similar tire model based on the stretched string concept and developed simple derivatives representing first order lag with a relaxation length and a gyroscopic couple coefficient as parameters. For the description of steady state slip characteristics empirical formula have been developed by Bakker and Pacjeka [1], using trigonometric functions, this model is known as Magic Formula.

The performance of braking system is an important consideration in the design of landing gear system. Luber et al [44] have shown in their experimental work that adjustable control of brake torque is a sensible way to improve aircraft ground handling and performance. Krüger et al [41] also mention the need of a good model of the anti-lock braking system dynamics. Yager et al [74] under the FAA/NASA friction program discuss the evaluation of friction measurements for different runway surfaces. General requirements of a good anti-skid brake system are described in an SAE (Society of Automotive Engineering) paper [66]. Unfortunately there is not much work published related to the brake dynamics and its effect on landing gear dynamics and aircraft in general. Although the know-how exists the brake manufacturers are not eager to share it. Since aircraft spends a good amount of time on the ground and related issues play an important part in modern aircraft design it is necessary to study the aircraft ground dynamics and the brake-gear interaction. One of the early investigations on brake-induced vibrations was reported by Edman [15]. The report contains both experimental and theoretical studies explaining the basic phenomena and pointing out the importance of design considerations. Only linear solutions were considered in this report, however, it was recommended that non-linear friction characteristics be included in future theoretical studies. The dynamo-meter tests revealed a connection between the chatter frequencies and the wheel rotation. Theoretically, decrease in chatter amplitudes were noticed for increase in strut damping, rolling radius, and total mass. Biehl [4] during the development of a digital program to simulate the DC-9 aircraft main landing gear found out that brake torque was the primary contributor to chatter and squeal vibrations. J. Enright [18] discusses a simplified technique for laboratory dynamo-meter simulation of landing gear-brake dynamics which enable it to be used as a matter of routine to study brake dynamics accurately. Hamzeh et al [27] discuss the friction induced instabilities in a simplified aircraft brake model. Denti and Fanteria [13] in their work discuss the effects of different tire models and brake on the longitudinal dynamics of aircraft landing gear. As far as simulation of landing gear dynamics is concerned two reports from the BF Goodrich Aerospace by Rook et al [55] and H. Vinayak et al [71] are state of the art in the area.

Some of the work published in the 1990ies can be considered as state of the art in the field of landing gear vibration studies. R. Black [6] has outlined an experimental and analytical program for prediction of airplane landing gear shimmy stability. Feld [20] studied the effect of Coulomb friction in landing gear oleo bearings and pressure seal. Li [43] in his work studied the landing gear instability and shimmy behavior of a cantilevered dual-independent-wheels nose-gear through modeling, analysis and simulation. He included important features such as free-play and nonlinear damping, dry friction between the piston and cylinder. He treated shock strut as a flexible beam. He showed that for fixed parameters, the gear may become unstable and develop shimmy as the taxiing velocity is increased beyond a critical point. Dr. Basselink [3] has studied the shimmy of aircraft main landing gears and the various parameters that affect the landing gear stability. For further research he has suggested to investigate the effect of braking on landing gear shimmy and other vibration modes. In a report published by Menasco Aerospace Ltd. [21] it is mentioned that the newer Computer Aided Engineering techniques made available by Multi-body Simulation combined with the import of flexible bodies using commercial tool can be used to investigate the problem of both landing gear shimmy and other vibrations.

1.3 Motivation and Solution Strategy

It is no longer sufficient to optimize the aircraft and components such as landing gear solely with respect to pre-defined critical conditions and the certification cases. If an aircraft is to be developed in a virtual design environment, then analysis, simulation, and testing tools must be capable to provide answers to complex interdisciplinary questions in this domain as well. The next generation of Computer Aided Engineering (CAE) tools in aviation will have to provide comprehensive interdisciplinary analysis and simulation capabilities. This is also true of Multi-body Simulation (MBS) of landing gear dynamics.

In aircraft ground dynamics, the loads applied on landing gear and airframe as well as the dynamic behavior of the aircraft itself depend strongly on the particular circumstances- the obvious influence factors such as aircraft configuration, attitude and sinking speed are important for a rough estimation, but for more detailed, reliable and comprehensive results the actual scenario has to be modeled and analyzed with a high degree of complexity, too [68]. As an example, the actual timing of lift dumper deployment during a landing has a significant impact on the applied loads: activated at the right moment during rebound after hard landing, the sudden loss of lift may cause the airplane to drop back into already compressed shock-absorbers, resulting in considerably higher loads on gear and airframe than at the landing impact itself. The effect is known, but being able to find the most critical condition(s) and to quantify the results is a challenging task.

So far, in most aircraft ground dynamic analysis, and in fact most aircraft developments in general, aircraft and landing gear designers have concentrated on certification requirements. If these requirements could be met, most critical conditions would be covered. New aircraft developments, however, may raise other, or additional, requirements:

- Economy: The certification requirements have been set up to ensure a high safety standard of aircraft, not to ensure best performance or low operating costs. In the case of aforementioned problem of lift dumper deployment, for example, it does not necessarily affect the safe operation of the aircraft. The fatigue performance of the landing gear will probably decrease, but with proper maintenance the components in question will be replaced in time to avoid any risk. Nevertheless, the impact on maintenance, and consequently operation cost can be significant.

- Applicability: For very large transport aircraft, some certification requirements do not represent reality as it is more or less based on empirical data. For the Airbus-A380, for example, the lateral loads due to turning are exaggerated, whereas other critical conditions like the torsion load on the main landing gear leg in sharp turn (e.g. during push-back from the gate) are not adequately covered.

- New Problems: Improvements in aircraft design, e.g. sophisticated lightweight constructions leading to increased structural flexibility, can cause new or so far uncritical phenomena to becomes an important issue. They have to be detected and counteracted as early as possible.

- New Concepts: The point stated above is even more valid for unconventional designs, e.g. blended wing bodies. Doubts about the validity of analysis results which cannot be crosschecked with empirical data and uncertainty about the regulations which will finally be applied are among the major concerns every time a new, unconventional configuration is being evaluated.

Advanced aircraft ground dynamics simulations will have to cover these additional requirements as well. Comprehensive and detailed physical modeling and analysis has to ensure that all realistic, "landing" conditions and scenarios can be simulated and evaluated.

Computer Aided Engineering(CAE) tools such as Flexible Multi-body Methods is a very good testbed for modeling a landing gear and complete aircraft system. Since the MBS tools such as SIMPACK are modeled as an open system they can accept inputs from many other standard software tools such as matlab etc. With the help of all these modules one can simulate important ground maneuvers in the pre-design phase to save high costs of flight-tests. Clearly MBS is the favored tool for analysis of the dynamics of the gear and brake system. It also allows concurrent engineering with other Computer Aided Engineering (CAE) tools such as Nastran which ensures accurate modeling for the purpose. It is a fast and efficient way of computation.

In this work, two different approaches of modeling a complex system such as landing gear are presented. The landing gear is modeled as a flexible beam as a full Finite Element mesh model is not necessary to study the dynamics of the system. In the first approach flexible landing gear is modeled assuming that the landing gear is on ground and the stroke (defined as shock-strut travel inside main-fitting tube) remains constant. In the second approach the translational degree of freedom is taken into consideration. These models are prepared in Nastran and inputed to SIMPACK using the interface called as FEMBS.

It is also important to model force routines to describe the accurate physical behavior of a system. For the same reason different force routines were also modeled. A hydro-pneumatic oleo which consists of spring, damper, and seal friction was modeled to describe the behavior of shock absorber between main-fitting tube and shock-strut. Tire dynamics play an important role in the vibrations of landing gear. Tire routine including lateral dynamics was also implemented. Various control algorithm were also implemented. These include a simple dynamic braking algorithm, braking based on slip-optimization principle, and modified braking algorithm which includes hydraulic dynamics.

With the help of all the modules important ground maneuvers are simulated. It was possible to simulate time-simulations and eigen-analysis. As a first task the models were tested to see if it is possible to detect landing gear vibrations. For this purpose different sensors were defined such as gear-walk and shimmy-sensor. It was possible to detect the vibrations be means of deflection at these sensors. As a next task different braking algorithm were also compared. It was found out that the modified braking algorithm works best from braking performance point of view and also passenger comfort. Finally a parameter study was conducted to find out the important parameters affecting the vibrations in the landing gear. It was found out that the most important parameter affecting shimmy was trail length, yaw stiffness, and lateral stiffness.

1.4 Contents

The aircraft ground dynamics and the related vibration issues are one of the key factors of a successful aircraft design. It has to be given adequate attention like other design considerations such as aerodynamics, flight mechanics, and propulsion. The research presented here deals with the important aspect of landing gear dynamics and its interaction with braking. It is also aimed to study and analyze the potentially unstable phenomenon such as gear walk and shimmy which may occur during ground maneuvers.

The work consists of six main chapters and they deal with the following -

Chapter 1 will give an introduction to the theme of the dissertation and brief explanation of the scope of the work.

Chapter 2 is intended to highlight fundamentals and background of the theme, aircraft ground dynamics, landing gear dynamics and its interaction with the brake system. It will also deal with the state of the art related to aircraft ground dynamics.

Chapter 3 explains the fundamentals of multi-body simulation and introduction to the MBS tool SIMPACK. It further explains why MBS is a favored tool for analysis of dynamics of a complicated system such as landing gear and brake interaction. It will also give brief introduction to the representation of deformable bodies in multi-body systems. Modeling strategy and the equations of motion will be explained in detail. It also explains how the flexible landing gear is modeled in Nastran.

Chapter 4 will explain modeling of the rigid landing gear with the use of SIMPACK. Modeling of other important modules such as use force routines and control algorithms.

Chapter 5 gives us the overview of braking algorithm and modeling of the same in MBS tool SIMPACK. It also delves into the advantages offered by a sophisticated 'real-life like' braking system.

Chapter 6 deals with the simulation cases and the results for important ground maneuvers such as rolling, braked run of two-mass model of a landing gear and complete aircraft. It also depicts simulation of vibrations in landing gear, namely gear walk and shimmy and if brake-gear interaction play a role.

Chapter 7 summarize the work and highlight the contribution to the field of landing gear dynamics simulation and brake-gear interaction. Suggestions for further work in this field will conclude the work.

Chapter 2

Background and State of the Art

2.1 Aircraft Ground Dynamics, Problem Definition

The early term for the wheels and support structure of an aircraft was "undercarriage". This is now termed as "landing gear" or the shortened version "gear" [29]. Although this later term indicates main function of the gear, namely containment of the landing impact loads and motions, it fails to indicate the other main function, namely the provision of means for the aircraft to maneuver on the ground, taxi and take off. Design for satisfactory operation in both landing and ground maneuvering involves compromises between the two.

One of the challenges the aircraft community facing today is landing gear dynamics, especially shimmy and brake-induced vibration. Although neither shimmy nor brake-induced vibrations are usually catastrophic, they can lead to accidents due to excessive wear, shorten life of gear parts, and contribute to the pilot and passenger discomfort [53].

History of aviation provides numerous examples of problems with landing gear dynamics, from the very beginning to the latest developments. Troubles of modern transport aircraft due to unforeseen or underestimated dynamic effects span from substandard ground handling qualities in crosswind conditions (eg.Boeing 767) over landing gear shimmy (eg. Fokker 100, Boeing C17 Globemaster) and brake chatter (eg. Fairchild Dornier 328 Jet) to vibrational problems due to airframe/landing gear or landing gear and -brake interaction. Since the emergence of jet fighter aircraft with increased take-off and landing speeds, this type of aircraft has been troubled by coupled heave-pitch oscillations on uneven surfaces. Because of larger wheel bases, high momentum of inertia and usually lower landing speeds, transport aircraft were less critical in this respect. Progress in performance and lightweight design, however has revealed an additional phenomenon: ground induced oscillations of the deformable, elastic airframe causing increased dynamic loads and partially violent local accelerations. Interaction of brake dynamics with landing gear may play an important role too.

One of the first civil transport aircraft which was seriously troubled by this effect is the BAC (British Aerospace Company) Concorde. Several factors contributed to this sensitivity; slender fuselage with pilots and first passenger rows situated way in front of the nose gear leg, take-off speeds (up to 215 kts in hot-and-high conditions) and high tire and oleo stiffness. The problem was eventually solved with a two-stage nose gear oleo with reduced stiffness at the MTOW (Maximum Take-Off Weight) working point. It was considered, at the time, to be a problem of this particular configuration. In the 1980ies, conventional transport aircraft designs turned out to become affected as well, eg. McDonnell Douglas (now Boeing) MD-90-30 and Airbus A340-600. It is to be expected that further progress in lightweight construction will increase the sensitivity of aircraft to this kind of vibrational problem as well as probably introduce new forms of dynamic interactions. It will be important for the economic success of future aircraft developments to detect and predict possible problems as early in the design process as possible - vibration interactions will hardly be "show stoppers" of a new design, but they are very expensive to overcome and usually lead to suboptimal solutions, as they are often discovered as late as in flight tests or even after entry-into-service of a new type, as it was the case for the A340.

Landing gear vibrations can be divided into two broad categories, namely shimmy and brake-induced vibrations. Shimmy may be caused by a number of conditions such as low torsional stiffness, excessive free-play in the gear, wheel imbalance, or worn parts. Friction-induced vibration includes conditions known as gear walk, squeal and chatter which are caused by characteristics of friction between the brake rotating and non rotating parts. Squeal refers to the high frequency rotational oscillation of the brake stator assembly whereas chatter and gear walk refer to the low frequency fore and aft motion of the gear. This will be explained in details in Section 2.2. Less known is the fact that the anti-skid braking algorithm may also play an important role in the landing gear vibrations. It is therefore important to study the effect of landing gear and brake algorithm interaction. The important structural parameters such as torsional stiffness, yaw stiffness, mechanical trail length, wheel-span, free-play, and brake dynamics play a role in stability and comfort which may be taken into consideration.

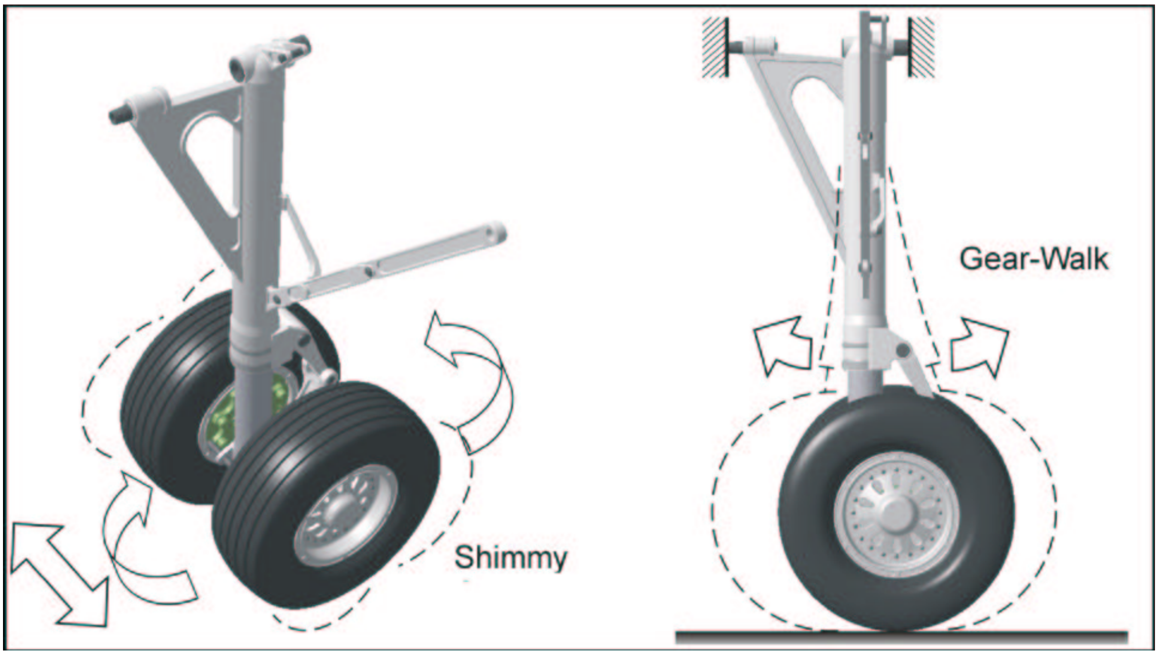


Figure 2.1: Shimmy and gear walk [42]

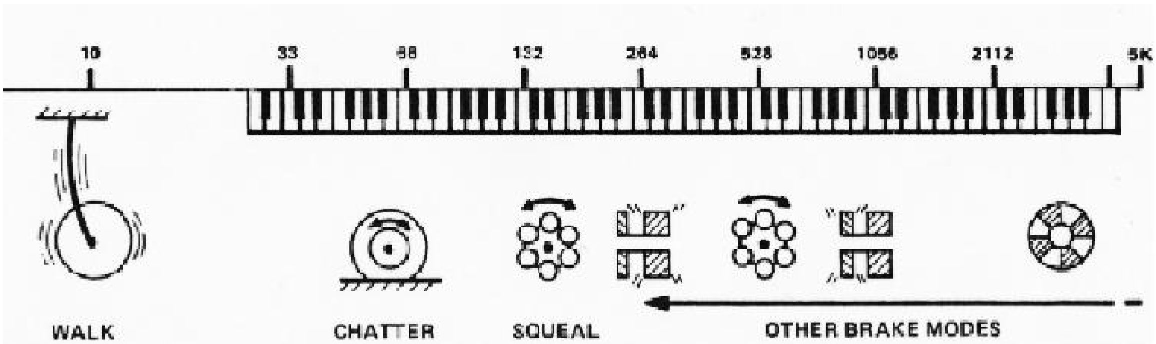


Figure 2.2: Vibration modes of a landing gear [18]

2.2 Friction Induced Vibrations in Landing Gear System

The aircraft landing gear, a complex multi-degree-of-freedom dynamic system may encounter vibration modes which can be influenced by brake frictional characteristics and design features [18]. As airplane gross weights are increased, the braking performance requirements have become more severe. The performance requirements include normal landing/refused takeoff braking distance limits, thermal requirements on the landing gear components, durability of friction material and overall weight considerations. Due to superior performance of carbon, increasing numbers of airplanes are using carbon brakes [49]. Although carbon has a higher specific heat capacity, a higher friction coefficient, is lighter in weight and has a better wear rate compared to steel, it is more prone to vibrations. Brake friction acts in the pitch-plane of the landing gear system, and so affects the stability of three pitch-plane modes of vibration as shown in Figure 2.2.

Brake Squeal can be defined as torsional vibrations of non-rotating components about the axle in the frequency range of 100-1000 Hz. The root cause of this mode is largely unknown, however, the erratic vibration phenomenon from flight test suggest that this mode is caused by the friction characteristics of brake material. it produces very high oscillatory loads on the landing gear/brake structure and can sometimes cause failure.

Brake Chatter is defined as the torsional motion of the rotating parts of the brake-wheel-tire assembly about the axle and against the elastic restraint of the tire. It is typically above 50 Hz and coupled with the squeal mode.

Gear Walk is defined as the cyclic fore and aft motion of the landing gear strut assembly about a normally static vertical strut center line. This motion is caused by tire-runway interface friction loads which deflect the landing gear. It may be sometimes induced by the anti-skid system and could cause passenger discomfort.

A valid landing gear simulation is one having the same dynamic response to brake torque as the actual gear. This means that the simulated gear must be designed to have the same equation of motion in its walk mode under the action of speed-dependent braking friction [18]. The traditional way to simulate the gear has been to use alternate structure, a dynamo-meter fixture such that one of its fundamental modes duplicates the dynamic characteristics of the gear walk mode of interest. In this paper, the flexible multi-body dynamics methods are used for the simulation of such an unstable and complex phenomenon during aircraft ground maneuvers to detect friction-induced vibrations in aircraft landing gear.

2.3 Shimmy

Since the introduction of pneumatic tires, automobiles and aircraft have suffered from unstable oscillatory swivel motions. The oscillations exhibited by the steerable wheels is popularly known as "shimmy". Shimmy is also the term used to describe the self-induced swiveling motion of the gear of an airplane. Pneumatic tires are used for obtaining better road-holding and comfort. The vertical and lateral elasticity of the tire introduce additional degrees of freedom of motion, which are coupled with the angular motions of the wheel about the swivel axis [50]. Such a coupling may lead to the occurrence of an oscillatory instability of the stationary rectilinear motion. In case of aircraft landing gear this may be caused by the landing gear structure and interaction of the brake and gear. These vibrations are usually in range of 10 to 30 Hz, and may assume grave proportions and cause failure of mechanical components or result in loss of control of the vehicle.

The problem has been studied since 1920ies. In 1925 Broulihiet [9] published his observations on the role of tire mechanics on shimmy behavior and are still followed today. He followed the problem of stability in a castering wheel and found out that it depended on the mechanical properties of the tire. Fromm [23] also studied wheel shimmy in automobiles and aircraft. He was one of the first to identify the vertical elasticity of the tire as the main contributions to the vertical displacement of the vehicle. His earlier investigations on rolling slip of deformable bodies led him to study the effect of side-slip or yaw of the rolling wheel due to lateral forces. He also recognized the similarities between vibration problems in automobiles and aircraft.

Tire mechanics played an important role to study the shimmy problem. Since the problem of shimmy and self-excited vibrations has existed for a long time, many theories on the elastic deformation of tires had been proposed. There was controversy over the advantages and disadvantages of these theories [11]. The tire theories were categorized into two groups based on the number of coordinates used to describe the tire deformation.

There was a group of investigators which tried to study the problem with the aid of tire models more of less based on string concept. Fromm [23] gave a simple theory where such model is investigated. In 1941 Von Schlippe presented his well known theory of the kinematics of a rolling tire. Later on, the work published by the duo of Schlippe and Dietrich [60] studied the effect of width of tire contact area. They made significant progress in defining the yaw angle and the swivel angle as arbitrary functions of time. Their tire concept was simplified as a thin band with lateral elasticity leading to simple expression for the forces and moments which is known as string theory. Smiley [64] gave a summary theory resembling the one-dimensional theory of

Von Schlippe. Pacjeka [51, 50] improved upon this model by using multiple stretched strings to simulate the width of the tire and non-stationary properties of the rolling tire are included. These methods are considered to be effective for low frequency applications. His method is particularly applicable to vibration problems of steering and suspension system of vehicles at high speed and frequency. Simple equations are derived that relate inertial forces to dynamic displacements and external ground forces to static displacements of the tire center plane. His analytical results compared well with experimental data. The other group considered the effect of kinematic behavior of tire on the overall system. This includes Moreland's [45] point contact theory that assumed the interaction between the ground and the tire could be treated as single point. This theory accounts for the effect of side force on the yaw angle of the tire and a time delay between the application of the side force and the steady state yaw. He theorized that the tire support flexibility was a more important consideration than the tire mechanics. He also stated that a shimmy theory based on the tire alone was insufficient and that torsional and lateral rigidities, the wheel moment of inertia, and the weight of the strut were also critical in defining system stability. Only a fairly complete model of the structure including the tire properties could properly evaluate the stability of the system.

There are different papers available on the realistic evaluation of the shimmy problem. R. Black [6] has outlined an experimental and analytical program for prediction of airplane landing gear shimmy stability. The method makes use of laboratory shimmy tests on a flywheel which simulates the runway and a landing gear mounting structure which simulates the fuselage. Differences between the laboratory tests and airplane tests are detailed. By using these differences, the prediction of airplane results is carried out by an experimentally verified analysis rather than direct application of the laboratory test results. The analytical model is outlined including the tire mechanics. Samples of correlation between analytical results and experimental results. He thus proposed a way to extend the laboratory results to the airplane system. Feld [20] studied the effect of Coulomb friction in landing gear oleo bearings and pressure seal. He demonstrated that in case of a dual-wheeled main landing gear Coulomb friction alone produces significant damping to shimmy under normal conditions. On the other hand in case of severe runway, tire or disturbing transient conditions , the magnitude of Coulomb damping is less than 1%. He suggested that the risk of potentially low damping from this source should be considered in designing of landing gear. Li [43] in his work studied the landing gear instability and shimmy behavior of a cantilevered dual-independant-wheels nosegear through modelling, analysis and simulation. He included important features such as freeplay and nonlinear damping, dry friction between the piston and cylinder. He treated shock strut as a flexible beam. He showed that for fixed parameters, the gear may become unstable and develope shimmy as the taxiing velocity is increased beyond a critical point. He also studied various parameters such as trail length, tire mass etc. on shimmy behavior.

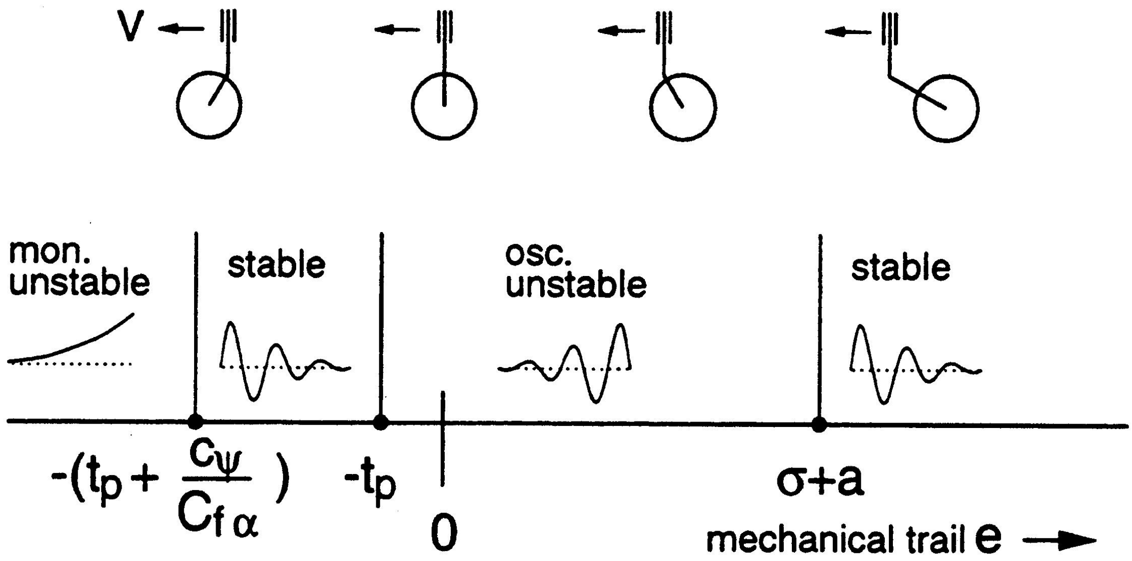


Figure 2.3: Shimmy stability against mechanical trail [3]

In March 1989, a left-hand main landing gear of a Fokker 100 aircraft failed in the landing gear at Geneva airport. Landing gear instability during or after landing gear impact was given as a possible reason. To investigate this hypothesis a mathematical model of the landing gear-tire combination was developed, taking into account only the main motions of the gear [70]. They concluded that any two-wheeled landing gear is unstable and shimmy damper is required to keep the vibrations below acceptable limits.

In his doctoral thesis Dr. Basselink [3] has studied the shimmy of aircraft main landing gears and the various parameters that affect the landing gear stability. He mentioned that the shimmy of landing gears is potentially dangerous and may result in sever damage to the aircraft. Therefore, it should be an important consideration in design of the landing gear. It is known that laboratory flight tests are not reliable enough to demonstrate shimmy stability. It is important to put effort in developing simulation models in order to make reliable stability predictions and understand the mechanisms governing shimmy. It is known from the analytical expressions that the system stability can be derived for the trailing wheel system with the lateral flexibility of the support. Figure 2.3 shows the stability region versus the mechanical trail. Two areas of stability exist,

- small negative trail combined with a low yaw stiffness and high lateral stiffness.
- large positive trail combined with a high yaw stiffness and low lateral stiffness.

For further research he has suggested to investigate the effect of braking on landing gear shimmy and other vibration modes. The main landing gear is equipped with disk brakes, consisting of a stack of rotors (connected to the wheel) and stators (connected to the axle) which are compressed by brake cylinders upon application of the brakes. In case of shimmy instability this may affect the lateral accelerations which may result in cyclic brake excitation due to lateral vibration of the landing gear. He has also studied the effect of free-play on landing gear vibrations. It is known that the free-play is a function of shock absorber closure. Section 4.2.4 explains in detail how it is modeled for this work. It can be represented fairly accurately by lumping the lateral free-play in side stay and yaw free-play as a translational degree of freedom at the apex joint. It is well known that the equivalent yaw stiffness will be reduced by introducing the yaw-free play; and for a gear with positive trail this may result in shimmy [3, 24, 43]. He further states that the fact that introducing lateral free-play may actually be beneficial of some landing gear configurations; see Figure 4.14. In a report published by Menasco Aerospace Ltd. [21] it is mentioned that the newer Computer Aided Engineering techniques made available by Multi-body Simulation combined with the import of flexible bodies using commercial tool can be used to investigate the problem of both landing gear shimmy and other vibrations.

2.4 Brakes

As the level of technology of human transportation has increased, the mechanical devices used to slow down and stop vehicles has also become more complex. Dr. F.W. Lanchester patented a design for a disk brake in 1902 in England. It was incorporated into the Lanchester car produced between 1906 through 1914. These early disk brakes were not as effective at stopping as the contemporary drum brakes of that time and was soon forgotten. Another important development occurred in the 1920s when drum brakes were used at all four wheels instead of a single brake to halt only the back axle and wheels such as on the Ford model T. The disk brake was again utilized during World War II in the landing gear of aircraft. The aircraft disk brake system was adapted for use in automotive applications, first in racing in 1952, then in production automobiles in 1956. United States auto manufacturers did not start to incorporate disk brakes in lower priced non-high-performance cars until the late 1960s. As with almost any artifact of technology, drum brakes and disk brakes both have advantages and disadvantages. Drum brakes still have the edge in cheaper cost and lower complexity. Drums are inferior to disks in dissipating excessive heat. Improvements in control have been made available with the application of Anti-Lock

Brake technology. Wheel sensors convey rotation speed of each wheel to a computer that senses when any of them are locked up or in a skid, and modulates individual wheel brake hydraulic pressure to avoid wheel skidding and loss of vehicular control.

Anti-lock Braking Systems (ABS) are used to prevent a vehicle's wheel from locking as a result of excessive operation of the service brake, especially on a slippery road surface. Thus lateral control on the wheels being braked is maintained even at full brake application or in panic braking situations to ensure the cornering stability and steer-ability of a vehicle to the greatest possible physical extent. At the same time, the objective is to optimize the utilization of the available adhesion coefficient between tires and the road surface or runway and thus vehicle retardation and stopping distance.

2.4.1 Description of an ABS cycle

Figure 2.4 shows the basic structure of ABS control. Accurate slip calculation is a key of ABS control. No matter what algorithm is applied, true velocity is always the basis of slip calculation. Wheel angular velocity can be measured easily and accurately by using wheel angular sensor. However, direct measurement of vehicle velocity such as optical correlation method or spatial filtering method, although available, is often too expensive and requires additional wiring, which makes the system more complex. Relying on an additional sensor also makes the system more prone to sensor failures, thus lowering system reliability.

Figure 2.5 shows an example of a control cycle with the most important control variables, wheel deceleration threshold ($-b$), wheel acceleration threshold ($+b$) and slip threshold λ_1 and λ_2. In case of impending wheel lock, the brake pressure of the corresponding wheel will be decreased, held during expected or measured wheel re-acceleration and subsequently increased in steps after re-acceleration. The cycle is started again if the brake force is still too high for the actual friction level. As the brake pressure increases, the wheel is progressively decelerated. At point 1 wheel deceleration exceeds a value that can not physically be exceeded by vehicle deceleration. The reference speed, which up to this point had been the same as wheel speed, now diverges and is reduced according to a fictitious vehicle retardation from point 2 (exceeding the threshold, $-b$) with a slower deceleration. The deceleration threshold $-b$ is exceeded at point 2. The wheel now moves into unstable region of the μ-λ slip curve at which point the wheel has reached its maximum braking force and any further increase in brake torque does not achieve any further deceleration of the vehicle but merely deceleration of the wheel. For this reason the brake pressure is quickly reduced and so wheel deceleration decreases. The time taken for wheel deceleration is determined by the hysteresis of the wheel brake and by the characteristics of the μ-λ

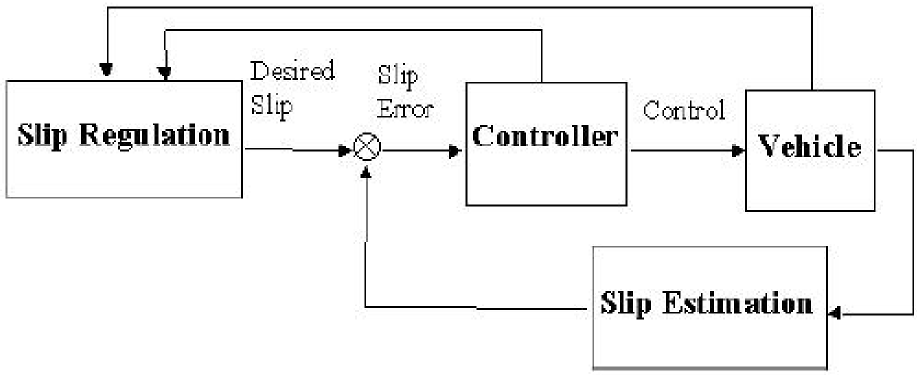


Figure 2.4: Basic ABS algorithm [30]

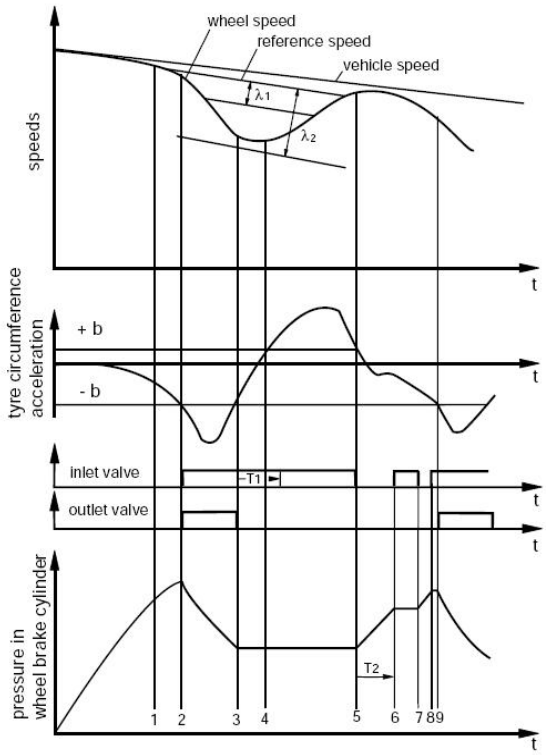


Figure 2.5: Description of an ABS cycle [25]

slip curve in unstable region. Only after the wheel brake hysteresis has been overcome does a continued reduction in pressure lead to a decrease in wheel deceleration. At point 3 the wheel deceleration signal $-b$ drops below the threshold and the brake pressure is held at a constant level for a set time $T1$. Normally, wheel acceleration will exceed the acceleration threshold $+b$ within this set time (point 4). So long as this threshold is exceeded, the brake pressure is kept constant. If (for example on a low-friction surface) the $+b$ signal is not generated within time $T1$, the brake pressure is further decreased by slip signal λ_2 is not reached. At point 5, the curve falls below the threshold $+b$. The wheel is now in the stable region of the μ-λ slip curve. Brake pressure is no rapidly applied for time $T2$ to overcome the brake hysteresis. The time $T2$ is fixed for the first control cycle. After the initial rapid phase, brake pressure is then increased more gradually by "pulses", by alternating pressure hold and pressure increase. The basic logic demonstrated in this example is not fixed at all; it adapts to the corresponding dynamic response of the wheel to varying coefficient of friction, i.e. it implements an adaptive type of system control. All threshold values depend on several different parameters, such as driving speed, vehicle deceleration, etc. The number of control cycles results from the dynamic response of the overall control system composed of the ABS control- the wheel brake -the road.

2.4.2 Brake-induced Vibrations: State of the Art

Technological advances in aircraft led to smaller brakes with more energy to dissipate, lighter shock struts with higher strength materials, and increased flexibility all of which increase the likelihood of vibrations of landing gear due to braking action. Brake-induced vibrations in landing gear may be induced for several reasons. The self-excitation of modes due to negative damping arises from variations in the coefficient of friction with instantaneous slip velocity. Forced oscillations are due to irregularities in the friction surfaces. Self-excited whirl vibration is caused by eccentricity of rotating and no-rotating brake parts. These are categorized as landing gear vibrations [67]. A uniform method of classifying brake characteristics was given in terms of coefficient of friction, dynamic variation of friction coefficient, wear variation, and torque versus pressure characteristics. It is further mentioned that self-excitation may be induced by large variations in the stiffness of brake components, poorly phased feedback in the anti-skid system, and tire lock-up characteristics. Solutions to these vibration problems included provision of basic aircraft parametric data from airframe manufacturers for analysis and testing. Data collection from flight testing is needed for skid control on wet and dry surfaces at shimmy speeds. Brake history and frequency and amplitude of vibration are desirable in order to characterize a pattern.

In a report published by Edman [15] a study of landing gear vibration due to brake chatter and squeal during taxi and landing was performed. The report contains both experimental and theoretical studies explaining the basic phenomena and pointing out the important design considerations. Static tests were conducted to determine parameters such as weight and mass moments of inertia, damping ratios, and spring rates that were needed for analytical studies. Dynamic tests were included brake and strut dynamo-meter testing that measured drag loads, brake pressure, wheel speed, side force, fore and aft motion of the axle, and angular acceleration of the axle. Taxi tests involved a number of relatively uncontrollable variables which is why it is difficult to achieve the same results with the dynamo-meter tests. Systems of individual masses, springs, and dampers were used to represent the landing gear to aid in studying the effects of friction characteristics of the brake on the dynamic stability of the gear. Only linear solutions were considered in this report, however, it was recommended that non-linear friction characteristics be included in future theoretical studies. The dynamo-meter tests revealed a connection between the chatter frequencies and the wheel rotation. Theoretically, decreases in chatter amplitudes were noticed for increases in strut damping, rolling radius, and total mass. Another effort to study landing gear chatter and brake squeal vibrations was at the Naval Laboratory during the development of a digital program to simulate the DC-9 aircraft main gear slowing to stop [4]. The analytical model represented the force and aft motion of the gear with accompanying rotational motion at the gear axle. Comparison of computed responses and measured data indicated reasonable simulation accuracy. The analysis showed that brake torque was th primary contributor to chatter and squeal vibration. Increasing the brake torque in combination with diminishing brake rotor to stator angular velocity instigated the vibration. This function effectively produced a negative damping that sustained or increased the vibration amplitudes. Attenuation methods included using a mix in the brake lining that ensured a flat brake torque function. Vibration absorbers were also suggested even though an excessive weight penalty existed for chatter vibration absorbers.

In his doctoral thesis Dr. Basselink [3] has studied the shimmy of aircraft main landing gears and the various parameters that affect the landing gear stability. For further research he has suggested to investigate the effect of braking on landing gear shimmy and other vibration modes. In a report published by Menasco Aerospace Ltd. [21] it is mentioned that the newer Computer Aided Engineering techniques made available by Multi-body Simulation combined with the import of flexible bodies using commercial tool can be used to investigate the problem of both landing gear shimmy and other vibrations. Landing gears vibration is potentially dangerous and may result in sever damage to the aircraft. Therefore, it should be an important consideration in design. It is known that laboratory flight tests are not reliable enough to demonstrate stability. It is important to put effort in developing simulation models in order to make reliable stability predictions and understand the mechanisms governing shimmy. Computer Aided Engineering(CAE) tools such as Flexible Multi-body

Methods is a very good testbed for modeling a landing gear system including brake algorithm. Since the MBS tools such as SIMPACK are modeled as an open system they can accept inputs from many other standard software tools such as matlab etc. With the help of all these modules one can simulate important ground maneuvers in the pre-design phase to save high costs of flight-tests. It is also possible to study the vibrations in the landing gear during these simulations, and interaction of brake and gear.

Chapter 3

Multi-body Simulation

3.1 Introduction

Originally, Multi-body Simulation (MBS) software was designed for the analysis of purely mechanical rigid body systems, sometimes added by force laws from other fields such as hydraulics or electronics, mostly included as source code. Since rigid body MBS is not relying on the exact structure and geometry of its components its main applications were principle dynamic investigations in the early development phase of a project. Today the request for the features of MBS-software is much more demanding. Modern MBS-software packages enable interdisciplinary modeling and analysis, either by own enhancements of the MBS functionality or via interfaces to other CAE tools or both. As a rule, the individual extensions of MBS programs are well adopted to the needs of MBS computation but limited in their facilities and performance. Interfaces to other CAE software on the other hand not only offer the entire possibilities and functionality of proven software tools but widely reduce the modeling effort as most of these models already exist, e.g. for CAD drawing or FEA stress analysis, only need appropriate conversion. Therefore Computer Aided Engineering(CAE) tools such as Flexible Multi-body Methods is a very good testbed for modeling a landing gear and complete aircraft system. Since the MBS tools such as SIMPACK are modeled as an open system they can accept inputs from many other standard software tools such as matlab etc. With the help of all these modules one can simulate important ground maneuvers in the pre-design phase to save high costs of flight-tests. Clearly MBS is the favored tool for analysis of the dynamics of the gear and brake system. It also allows concurrent engineering with other Computer Aided Engineering (CAE) tools such as Nastran which ensures accurate modeling for the purpose. It is a fast and efficient way of computation.

3.2 Why Multi-body Simulation

Several computer programs for kinematic and dynamic analysis were developed since the late 1960s. Descriptions, benchmark tests and summaries can be found in [36, 59]. The rapid progress in computer- and software technology is the main driver for changes in the strategies and methods used in aircraft design. The impact is ubiquitous. The main progress has been the development of a multitude of software tools for aircraft analysis and design. In last years, their applications have developed from stand-alone tools to solve specific, usually mono-disciplinary problems in the development of an aircraft [68]. A vast amount of activities in research and software development have been devoted to create interdisciplinary links and multidisciplinary analysis and optimization capabilities. This is denoted as CE (Concurrent Engineering), a systematic approach to the integrated, concurrent design of products and their related processes, including manufacturing and support. The concept of CE was made possible by the progress in computational engineering. CAE tools like CAD (Computer Aided Design), FEA (Finite Element Analysis) or CFD (Computational Fluid Dynamics) are well known examples for engineering software which provide the necessary analysis power in the specific engineering discipline. The specific CAE software tools apply different modeling strategies. Each discipline focuses on some aspects of the system. Models of different systems may have a certain redundancy, which can be used for coupling, eg. CAD can provide other tools such as FEA or MBS with geometrical and mass data. Aviation has always been a driver toward more sophisticated analysis methods and design strategies. The concept of CE was quickly adopted in major aircraft development programs. The Boeing 777 was considered and also marketed as the first aircraft completely designed in the computer.

The aircraft landing gear is a complex multi-degree-of-freedom dynamic system which may encounter vibration modes which can be influenced by brake frictional characteristics and design features [18]. Although equations for representing various parts of a landing gear are well established, solving the problems manually with mathematical programs can be slow and laborious. Simplifications made to reduce problem size may introduce inaccuracies such that a design modification to correct a problem in one area causes unforeseen vibration in other parts of the structure. In many cases, vibration problems may not be uncovered until physical prototypes are built and tested, adding considerable time and expense to the product development cycle.

After the governing equations of motion have been derived by manual or symbolic computation methods, the engineer is still facing the problem of obtaining a solution of the differential equations and initial conditions. Since these equations are highly nonlinear, the prospect of obtaining closed-form solutions is remote, except in very simple cases. With the advent of digital computers, engineers began to use the computer with available numerical integrating methods to solve their equations of motion. This, however, still involved a substantial amount of time and personnel for deriving equations of motion and writing ad hoc digital computer programs to carry out numerical integration [73].

The objective of computational methods in kinematics and dynamics is to create a formulation and digital computer software that allow the engineer to input data that define mechanical systems of interest and automatically formulate governing equations for kinematics and dynamics, solve them and provide computer graphics output of results of simulations to communicate results to the designer. Further developments in computer simulation techniques have lead to different approaches [41].

- Custom-made simulation software solves specific problems of aircraft ground dynamics. In most cases, it is in-house software of aircraft manufacturers or landing gear suppliers. Some of them in modular form (eg. airframe structure, landing gear model, numerical treatment, solution analysis) and can be assembled to different solution sequences. Examples of custom-made codes are GRAP and SD-approach (BAe Systems, Stirling Dynamics Ltd.) [10].

- Commercial engineering software tools usually offer improved handling qualities, more detailed documentation and high degree of continuity. In general, they represent the latest state-of-the-art in their specific discipline:

  - General simulation environments are widely used in industry for various applications. These tools, eg. MATLAB Simulink [47, 5], MATRIX$_X$ [28], Systembuild, offer easy-to-use possibilities for conventional(linear) system analysis.

  - Engineering software packages specialized on system dynamics analysis provide at least the same functionality as offered by custom-made applications. The most common tool in this respect is multi-body simulation software, eg. SIMPACK [56], DADS [65], or MSC.ADAMS [48], which can be used for vary detailed, nonlinear simulation of complex scenarios.

With increasing importance of an aircraft's dynamic behavior on the ground and growing complexity and interdisciplinary of the problems to be solved, the use of specialized commercial simulation tool is clearly favored in industry and research. Today, almost all major aircraft and landing gar manufacturers use one of the major MBS software packages for their ground dynamic analysis. One of the major application of MBS simulation in this area is landing gear design and its interaction with other systems such as brake. The ability of MBS tool to provide a virtual testbed for realistic, in-depth simulation of an aircraft's dynamic behavior is used to investigate new and improved concepts or to tackle prevalent problems and develop fundamental solutions. Examples are the evaluation of the ground dynamics of very large aircraft [19], or the investigation about the benefits of semi-active landing gear shock absorbers to damp the resonance effects during ground run [73, 38].

3.3 Multi-body Simulation

3.3.1 Overview

Multi-body simulation codes are efficient CAE tools to simulate the linear and non-linear dynamic behavior of mechanical and mechatronic systems. An important part of this wide area is the system dynamics of the vehicles [61]. In practical applications, MBS can be regarded as a "virtual testbed" for these systems. Behavior and performance of the entire system, or of its major components, are being evaluated in a virtual environment often long before the first prototype of the system is ready for field tests. In this respect, it represents a central tool in a virtual design environment. Figure 3.1 shows how MBS is being embedded into the world of CE by interfaces to other CAE tools, thus forming a specific part of CE network. MBS systems usu-

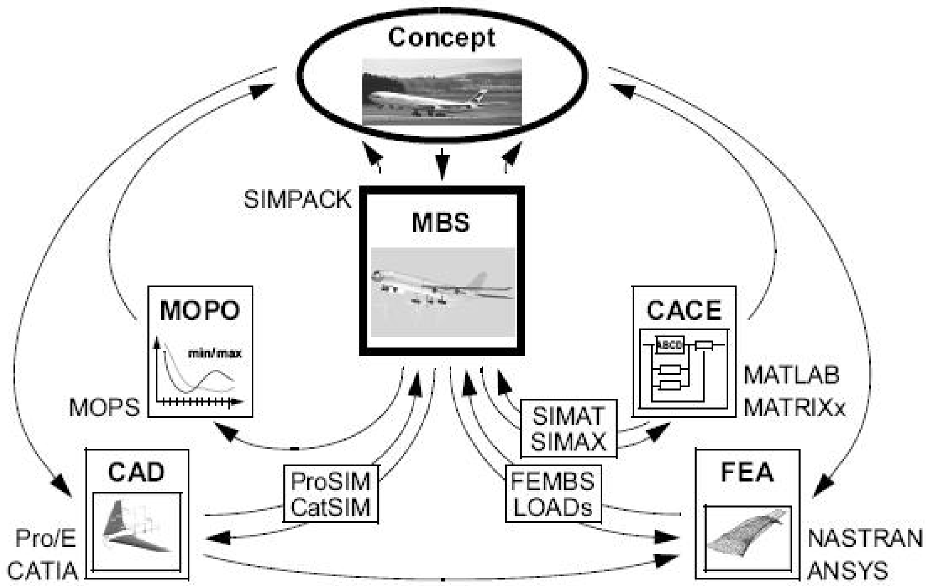


Figure 3.1: Multi-body simulation in concurrent engineering environment [68]

ally consist of a multitude of distinctive bodies which undergo large translational and rotational motions relative to their surrounding area as well as to each other. Similar to the components of a real mechanical system, connections to other parts of the system and applied forces influence the motion of an MBS body. The bodies may additionally be subjected to relatively small, elastic deformations. Accordingly, multi-body system dynamics may be defined as "the dynamic analysis of systems of interconnected bodies undergoing general translation and rotational motions [61]".

The discipline of multi-body simulation descends from the classical mechanical problem of translational and rotational motions of rigid bodies. *Newton, D'Alambert, Euler and Langrange* created the basis for deriving the equations of motion of multi-body systems. The rise of mechanical mechanism and machinery in the 19th century stimulated interest in kinematics and, to a lower extent, system dynamics of these mechanisms, but in general analysis capabilities remained limited to linear (or non-linear) systems undergoing small or planar motions or vibrations.

In the 1960ies, the situation changed. The need for more capable analysis of dynamic systems, eg. for nonlinear motion of high speed mechanisms or of space craft, boosted the activities in this area - supported by the fact that with the rise of computational abilities, efficient analysis of complex systems became feasible. The first "general purpose" multi-body programs were constructed [31, 54]. Its multi-body formalisms already allowed generating and integrating the equations of motion automatically from an input data set defining the geometrical and mechanical properties of the bodies, their interconnections and the system state at initial time [61].

The 1980ies saw the first commercial products established on the general engineering market. From then on, new multi-body formalisms, eg.O(N)-algorithms [7], generating the equations of motion in explicit or in residual form [17], drastically cut down the computational effort. Various numerical integration algorithms were developed or incorporated to insure stable and adequate numerical computation. Besides time integration, MBS code offer a variety of special numerical analysis methods, in particular for linear system analysis (linearisation, eigenvalues, root locii, frequency response, stochastic analysis in frequency and time domain), stationary solutions (equilibria, nominal forces) and kinematic analysis. Graphical User Interfaces (GUI) for model setup and evaluation simplified the use of dynamic analysis and reduced the sources of error in model set-up and interpretation of results. An overview of multi-body codes can be found in [59, 36].

3.3.2 Fundamentals of Multi-body Simulation

The method of multi-body simulation supports primarily the analysis of the motion, ie. kinematics, kinetics and dynamics of mechanical and mechatronic systems.

After input of the describing model data (eg. system topology, mechanical properties of bodies and joints, applied external forces and moments, initial values), MBS codes automatically generate the equations of motion of the model as a nonlinear set of equations, generally in the form of a system of Ordinary Differential Equations (ODE) or Differential-Algebraic Equations (DAE). A variety of optimized solvers are available to generate solutions numerically. A MBS system consists of two types of elements: Bodies and Connections. Bodies may be rigid or deformable, whereas connections may be kinematic (joints) or kinetic (forces).

Bodies

Rigid bodies have a simple structure: they are characterized by a reference frame, their mass and inertia tensor, and usually additional frames (markers) as attachments for force elements or joints to other bodies. Although bodies may boast a detailed, perhaps CAD-generated graphical representation of model set-up and results, this rudimentary data set is sufficient to represent the respective (rigid) body in MBS analysis.

In principal, this set-up of an MBS system containing deformable bodies does not differ from a purely rigid MBS system. For a deformable body, the MBS system receives an additional time dependency - the elastic deformation of the body. In general, elastic bodies are modeled under the assumption of small elastic and reversible deformation which usually derive from the linear superposition of pre-calculated mode shapes.

Joints

Joints are assumed as ideal, backlash-free and weightless connections between bodies (or frames). They reduce the number of degrees of freedom, forcing the bodies of an MBS system to motion sequences which would not occur without them. Thus, joints have to apply reaction (or constraint) forces acting orthogonal to the motion planes defined by the constraints. The reaction forces restrict the motion envelope of the system so it conforms with the geometric boundary conditions of the system.

Two different types of joints can be distinguished as: "normal" joints are connecting links in MBS systems with tree-like topology, respectively are those links in a system which connect the 'from body' to a body of higher topology level (so this system would have a tree like structure of only this type of joints were present), whereas links which close a kinematic loop are, obviously, called loop-closing joints, see Figure 3.2.

Force Elements

Force elements apply external or internal forces and torques in the system. They may depend upon the state of the system, eg. the distance between two points, and upon time. Force elements do not affect the degrees of freedom of the system, but may introduce additional states, or boundary conditions, to the differential equation system of the MBS model.

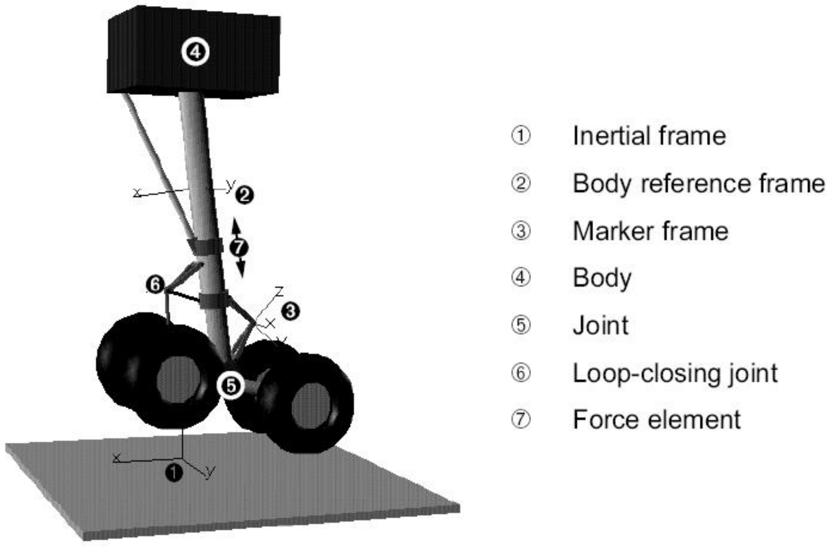


Figure 3.2: Elements of MBS model [68]

Analytical Techniques

Modern MBS tools offer a multitude of methods to analyze and simulate the generated MBS system. An overview of analysis features can be found in [57]; the most important are:

- Static Analysis - includes the computation of quantities of interest, eg. positions, applied or constrained forces or other measurements, in static equilibrium or quasi-static states.

- Kinematic Analysis - serves for system assembly, eg. computation of consistent initial conditions of closed-loop systems on position, velocity, acceleration level, and simulation of the kinematic behaviors (forward and inverse kinematics) of a model.

- Linear System Analysis - linearizes the equations of motion numerically, which opens up the entire range of linear system analysis methods, such as convariance analysis, computation of eigenvalues and eigenvectors or root locii analysis.

- Nonlinear Dynamic Analysis - delivers a numerical solution of the equations of the motion at distinct time steps. A variety of integrators are available to efficiently treat the problem in question, eg. with respect to numerical stiffness and state or time discontinuities.

3.3.3 Multi-body System Coordinates

In general, two basic approaches to define the multi-body system exist [63]:

- In the first approach, the configuration of the system is identified by using a set of Cartesian coordinates that describe the locations and orientation of the bodies, resulting in six coordinates for each body to account for the six degrees of freedom of rigid body motion. The connection between bodies are introduced by an additional set of nonlinear algebraic constraint equations, thus forming a set of DAEs. This approach is often referred to as using *absolute coordinates.*

- The second approach, with its *relative coordinates*, account for the reduction of degrees of freedom because of joint connections between bodies from the outset: it describes the location and orientation of a body in reference to the 'from-body', ie. the neighboring body which has a lower level in the system's kinematic topology. Thus, only the actual degrees of freedom of the connecting joint, and consequently of the body in question, are added to the system.

Depending on the application, the relative coordinates approach will generate a significant smaller set of equations. This advantage is at least partially impaired by the comparatively increased complexity of the equations of motion and more complex generation of Jacobian matrix. Both approaches are realized in commercially available MBS software packages; advanced methods are employed to accelerate the MBS analysis, eg. sparse-matrix algorithms for efficient handling of the large system matrices (esp. the Jacobian matrix) generated with absolute coordinates, or scanned-Jacobian techniques to accelerate the evaluation of Jacobian in relative coordinates. A detailed comparison between both approaches can be found in [56].

3.3.4 Multi-body System Formalisms

The motion of a multi-body system with the mass matrix $\mathbf{M}$ can be described by its generalized coordinates $\mathbf{q}$, velocities $\dot{\mathbf{q}}$ and accelerations $\ddot{\mathbf{q}}$

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}(\mathbf{t}) = f(\mathbf{q}, \dot{\mathbf{q}}, \lambda, \mathbf{t}) - \mathbf{G}^{\mathbf{T}}(\mathbf{q}, \mathbf{t})\lambda \tag{3.1}$$

$\mathbf{f}$ is the vector of applied and gyroscopic forces and $\mathbf{G}^{\mathbf{T}}\lambda$ represents the constraint forces. The constraint matrix $\mathbf{G}$ defines the restrictions which enforce a system motions consistent with the kinematic constraints, and the vector of Lagrangian multipliers λ contains the magnitude of constraint forces.

The equations of motion can be solved by two different approaches [56]:

- Classical formalism reduce the equations of motion by mechanical principles, eg. the principal of virtual work. With the additional information that the constraint forces act orthogonal to the unconstrained motions of the system, they separately compute mass matrix, constraint matrix and the applied forces. The computational effort increases at least with the power of two in respect to the degrees of freedom of the system.

- $\mathbf{O}(n)$-formalisms account for the orthogonality of constraint forces and unconstrained coordinates at a local joint rather than for that of the entire system. Additionally, they exploit the kinematic structure of the system. As a result, the equations of motion can be generated explicitly in the form

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \lambda, \mathbf{t}) - \mathbf{G}^{\mathbf{T}}(\mathbf{q}, \mathbf{t})\lambda) = \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}, \lambda, \mathbf{t}) \tag{3.2}$$

- For $\mathbf{O}(n)$-algorithms, the computational effort increases only linearly with the degrees of freedom. Further reduction can be achieved by residuum formalisms, which generate the equations of motion for implicit integration algorithm, using information about the characteristics of the terms which are exerted by the integrator.

3.3.5 Numerical Integration

In the equations of motion, all unknown variables (absolute acceleration, Lagrangian multipliers etc.) appear in linear form. The transformation of the kinematic state variables $\mathbf{z}$

$$\mathbf{x} = (\mathbf{z}^{\mathbf{T}}, \dot{\mathbf{z}}^{\mathbf{T}})^{\mathbf{T}} \tag{3.3}$$

delivers a set of first order differential equations of the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{t}) \tag{3.4}$$

As we know the state $\mathbf{x}$, the first derivative of $\mathbf{x}$ with respect to time, $\dot{\mathbf{x}}$, and consequently all other unknowns, can be computed. Choosing an initial condition

$$\mathbf{x_0} = \mathbf{x}(\mathbf{t} = \mathbf{t_0}) \tag{3.5}$$

allows to execute a numerical time integration of the equations of motion, delivering the system's behavior over time. The choice of the employed integration algorithm is the system. The precision is defined by users requirement. Stiffness and stability are influenced by physical properties of the system, the modeling approach and the characteristics of the integration method. Additional factors such as state-dependent discontinuities may further limit the range of applicable integrators. Table 3.1 gives an overview of MBS integration methods, [56].

Integrator	**Method**	**Area of Application**
DOPR156	Runge-Kutta method	non-stiff, smooth models
RK Bettis	Runge-Kutta method	non-stiff models with splined model parameters
LSODE	multistep method	stiff systems, systems with elastic components
LSODA	multistep method	systems with state-dependent stiffness
LSODAR	multistep method	systems with state-dependent stiffness
LSODART	multistep method for DAEs	systems with closed loops, with or without discontinuities
RADAU 5	backward Euler method	systems with closed loops, with highly oscillating components
EXPEUL	forward Euler method	for test and comparison
IMPEUL	backward Euler method	for test and comparison

Table 3.1: Numerical Integration Methods for MBS Equations of Motion

3.3.6 Multi-body Simulation in Aircraft Ground Dynamics

The technology of multi-body simulation with its associated software tools has been developed to analyze complex, arbitrary mechanical and mechatronic systems. The dynamics motion of a free-flying aircraft is apparently dominated by the movement of one body with its six rigid body degrees of freedom - a system which does not require the particular capabilities of sophisticated multi-body codes.

It has to be observed that the emphasis of aircraft ground dynamics analysis lies on the correct representation of the dynamics of the vehicle, not the flight mechanics. Aircraft ground dynamics has been dealt with by using multi-body simulation for a long time and with excellent results. This is not astonishing as the dynamics of aircraft landing gears and their interaction with the airframe represent a somewhat "classical" application of multi-body simulation, quite similar to those areas of vehicle dynamics such as automotive or wheel/rail, [35]. In fact, MBS represents a major tool for aircraft ground dynamics analysis and evaluation, ground loads analysis and airframe and landing gear certification [19, 32, 33, 34, 41, 38, 40, 68, 71, 73].

Thus multi-body simulation serves in wide range of applications fields in aircraft ground dynamics analysis - throughout the aircraft's design process. Among them are:

- overall dynamic behavior, eg. landing impact, high-speed rolling;
- ground loads on airframe and landing gear;
- handling qualities on/near ground;
- ground handling, eg.push-back, sharp low-speed turning maneuvers;
- rough or unpaved runway performance;
- shimmy analysis;
- safety issues and other case which are difficult or hazardous to examine in field test;
- certification analysis and
- analysis of unconventional configurations where no data is available to serve as a reference for heuristic approaches.

Originally considered as a tool for the early design stages, MBS is now being employed throughout the aircraft development cycles.

3.4 SIMPACK : A Brief Description

The objective of computational methods in kinematics and dynamics is to create a formulation and digital computer software that allow the engineer to input data that define mechanical systems of interest and automatically formulate governing equations for kinematics and dynamics, solve them and provide computer graphics output of results of simulations to communicate results to the designer.

Several computer programs for kinematic and dynamic analysis were developed since the late 1960s. Descriptions, benchmark tests and summaries can be found in [36, 59]. The software package used for this study is SIMPACK®, a state-of-the-art MBS-program developed by German Aerospace Center (DLR) which is now available as commercial multi-body simulation tool. It offers a fast algorithm for non-linear multi-body systems(MBS) combined with a multitude of libraries for modeling, computation and post processing [37]. The equations of motion are formulated in terms of relative coordinates. An efficient algorithm directly yields the explicit state-space form for tree-configured MBS with a minimal number of operations, for more details, See Section 3.3.2. SIMPACK is modeled as an open system they can accept inputs from many other standard software tools such as matlab etc. With the help of various inputs one can simulate important cases in the pre-design phase to save high costs of flight-tests. It is a medium level of complexity tool and a very good testbed for modeling a landing gear system including brake algorithm.

3.4.1 The fundamentals of SIMPACK

The basics of SIMPACK as of any MBS-software are its *multi-body formalisms*, ie. the algorithms which automatically generate equations of motion. In SIMPACK CPU-time saving *O(N)*-algorithms (the number of iterations grow only linearly with the number of degrees-of-freedom) establish the equations of motion in explicit or in residual form [37, 17]. The equations of motion for the MBS are setup in the form of ordinary differential equations (ODE) or (particularly in case of closed-loops) in differential-algebraic for (DAE). Adequate solvers have been developed for numerical integration, See Table 3.1.

Figure 3.3 shows the main functional modules of SIMPACK. Extensive libraries of coupling elements like joints and force elements as well as excitations aid the engineer in setting up a model. User routines can be introduced to extend the modeling options. User defined applied forces are introduced as simple force laws or as first order differential equations. For easy modeling, the MBS-library contains a large number of elements which are needed for modeling a typical MBS system such as joints, force elements, input functions and substructures like wheel suspension, complex kinematic joints, friction and impact elements as well as general use functions for new and special purpose elements to be generated by individual user. Parallel to the the model setup all the data are generated which are needed for the physical and graphical description of the MBS-model. The model generation is directly linked to the visualization and animation options, allowing user to view and check the system and simplifying the interpretation of the results. It is also possible to use analysis features such as *parameter variation function* which can be used for design and sensitivity calculations. It also provides additional control and vehicle functionalities. The control element makes it possible to use sensors to measure system states, absolute or relative kinematics, forces and output of feedback loops, discrete controllers to A/D- or D/A converters, and Actuators to have nonlinear characteristics. For analysis and design of vehicles' lateral and longitudinal dynamics it also provides various tire models based on different approximation methods (Pacjeka Magic Formula, Lugnar tire, HSRI, Delft tire etc.). The MBS-model for further SIMPACK analysis is generated by a graphical interface. SIMPACK has been designed as an open system which accepts various kinds of inputs from external standard software products. Interfaces have been established which allow links and transfers of various depth to and from other tools.

Linear Analysis

File transfer of linearized system matrices "A, B, C, D" established by SIMPACK to linear analysis and design tools of the MATLAB type for further analysis within the receiving software packages.

Elastic Structures

Two major options are available to analyze flexible mechanical systems: the finite element method (FEM) and the multi-body system approach. A simulation based on finite element models is, despite of the labor for setting up the model data, straightforward, the corresponding codes are well developed, and include linear and nonlinear theory of elasticity. Unfortunately, dynamic analysis with FEM-codes is very time consuming. In many applications one is confronted with system models, in which the deformations of the flexible bodies are small but superimposed of a large reference or 'rigid body motion'. In multi-body systems one exploits this fact to reduce the computational burden for such applications by linearizing the equations of motion assuming small deflections. Using relative variables to represent the reference motion and applying *O(N)*-formalisms [62], MBS-codes provide an efficient alternative for system analysis. In this work multi-body system approach has been preferred, See Section 3.5.3.

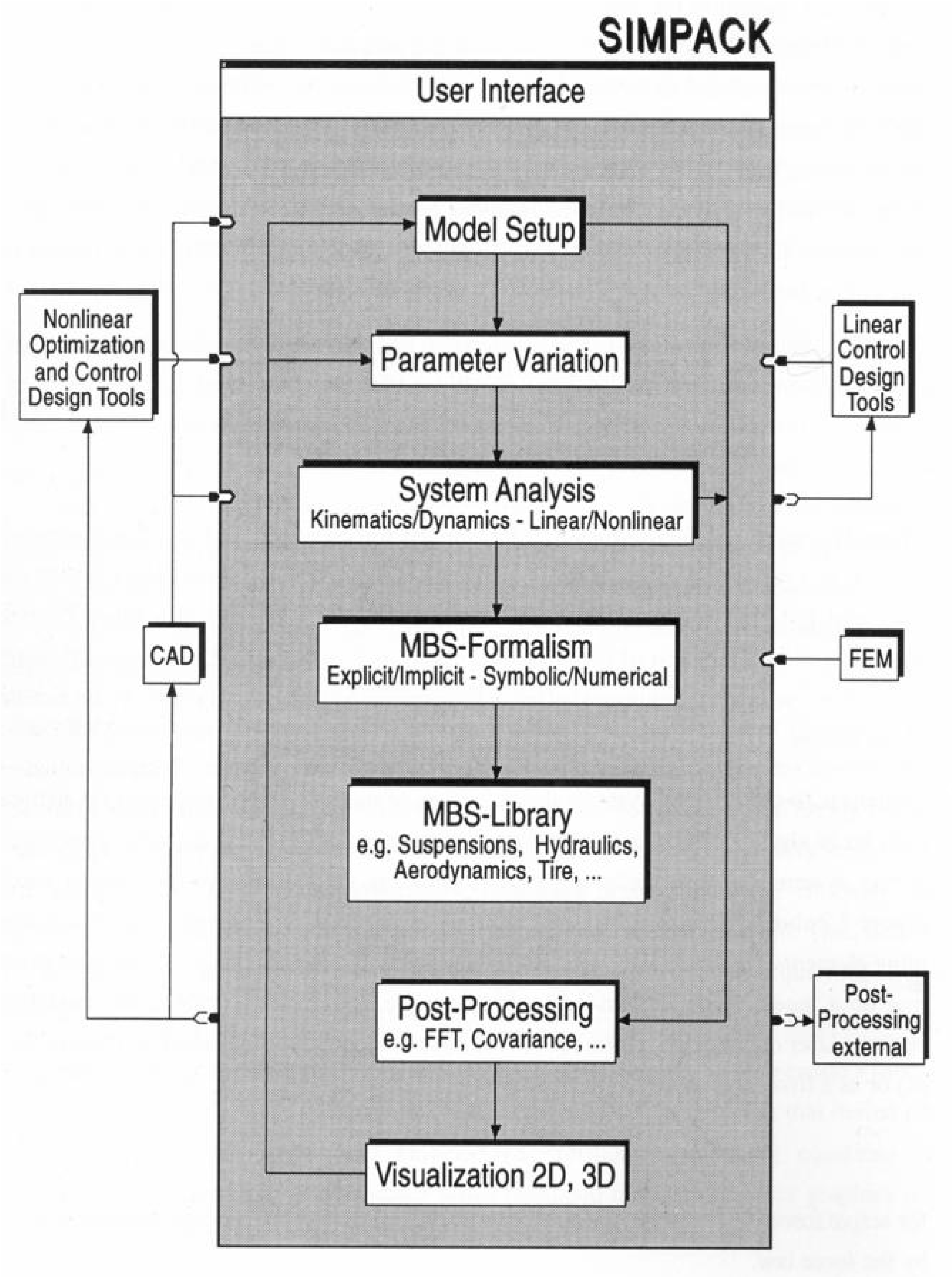


Figure 3.3: Functional modules of SIMPACK [68]

3.5 Flexible Bodies in Multi-body Simulation Systems

Multi-body codes are efficient tools to simulate the nonlinear dynamics behavior of flexible multi-body systems undergoing large overall motions accompanied by small elastic deformation. FEA analysis has its focus on the deformation of the structure whereas multi-body simulation is used to study the dynamic behavior of overall system. Applications are usually characterized by a quantity of bodies undergoing large, nonlinear transformations or rotations. MBS modeling and solution strategies are optimized to deal with this specific problem where systems undergo large deformations or motions with small elastic deformation. The representation of deformable bodies in MBS is adjusted to the requirements and characteristics of this kind of application [56, 62, 68, 72].

Most approaches represent the time-dependent movement of a deformable body by a large rigid body motion (the body being represented by its body fixed reference frame) which is superimposed by small deformation. The unconstrained gross motion of the body reference frame, with its six degrees of freedom, can be described by a set of six independent second-order differential equations of motion. The exact configuration of deformable body itself can be identified by an infinite number of elastic coordinates. The computational method which is used for flexible body analysis, FEA, still introduces a large number of degrees of freedom (10^6 DOFs for a typical application) - too many to solve reasonably in a complex multi-body system. Approximation methods are needed to reduce the number of elastic degrees of freedom in an appropriate way, ie. to account for those effects of body deflection that have significant influence on the dynamics of the system.

A common strategy is to use Berboulii's principle of separation of variables to describe the state- and time-dependent displacement field of the deformable body, by state-dependent base functions and time-dependent elastic coordinates. Thus, the original set of partial differential equations representing the dynamics of the deformable body is converted; the equations of motion now form a set of ordinary differential equations. Approximation methods, eg. Rayleigh-Ritz or Galerkin methods [8], can be employed to reduce the system to a finite number of coordinates. With the introduction of base functions, the equations of motion of the deformable body contain state-independent volume integrals which are responsible for the coupling between rigid body motion and elastic deformation. They can be computed prior to integration of the equations of motion itself.

The quality of the solution depends on the quality of the base functions. Of the three different types of base functions, the use of eigenfunctions has offered advantages in technical applications. Eigenmodes usually form the core set of base functions, possibly enhanced by staticmodes or inertia relief modes, eg. to account for local deformations due to large point loads on the structure or geometric boundary conditions.

Various methods exist to process suitable base functions, to optimize the approximation to the particularities of the application and to compensate for errors; an overview is given by Sachau [58]. The approach used in this work is based on the works of Rulka [56], Wallrapp [72], and is based on eigen- and staticmode analysis performed in nonlinear FEA software tools using the consistent mass approach of FEA formulation to integrate the volume integrals of the modal coefficient matrices.

The modal representation of deformable bodies in MBS implies several presumptions and model conditions which have to be observed.

- The elastic body may undergo large overall body motion, as indicated by the displacement vector $\mathbf{r}_{IB}$ in Figure 3.4. This motion may be accompanied by small elastic deformations $\mathbf{r}_{def}$ of the body, which are given in respect to the reference location on the undeformed body $\mathbf{r}_{BV(0)}$.

- The state of deformation of a body is measured in its reference system. The reference configuration of the body is undeformed state which has to be unequivocal and time-independent. This precludes the consideration of elastic bodies with material creep effects.

- The body is not exposed to internal force effects, eg. on polarized materials in electromagnetic fields.

- The state variables of deformation (elastic coordinates) are assumed to be small on position and velocity level; on acceleration level, however, they may be significant. Thus the mass matrix of the deformation body remains symmetric.

3.5.1 Representation and Kinematics of the Flexible Body

The approach of separating the large gross motion of the body from its small elastic deformations, see Figure 3.4, allows to write the absolute position $\mathbf{r}_{IV}$ of a volume element dV as

$$\mathbf{r}_{IV} = \mathbf{r}_{IB} + \mathbf{r}_{BV} \tag{3.6}$$

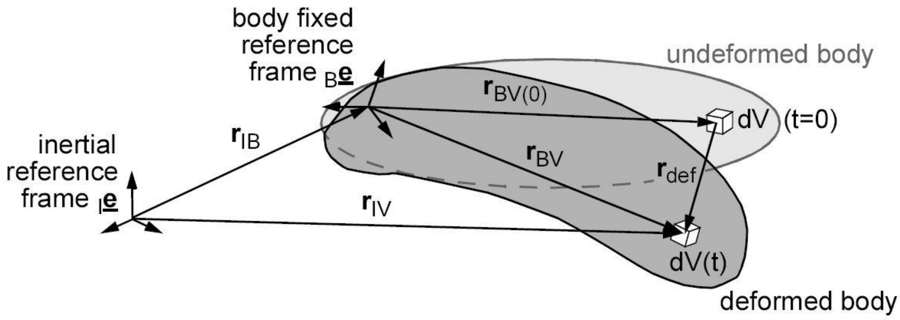


Figure 3.4: Separation of global motion and deformation

The state of deformation of a deformable body can be described by specifying, for every volume element dV of the body, the position vector of the volume element $\mathbf{r}_{BV}$ with respect to the position vector $\mathbf{r}_{BV(0)}$ of dV in the undeformed reference configuration and time t,

$$\mathbf{r}_{BV} = \mathbf{r}_{BV}(\mathbf{r}_{BV(0)}, t) \tag{3.7}$$

thus introducing the displacement vector $\mathbf{r}_{def}$:

$$\mathbf{r}_{BV}(t) = \mathbf{r}_{BV(0)} + \mathbf{r}_{def}(\mathbf{r}_{BV(0)}, t). \tag{3.8}$$

Introducing this approach into the equations of motion generates a set of partial differential equations; a general, explicit solution is apparently impossible. Bernoulii's principle of separation of variables allows converting the equations of motion to a set of ordinary differential equations. For the deformation vector $\mathbf{r}_{def}$, this step yields

$$\mathbf{r}_{def} = \mathbf{z}_E(t). \tag{3.9}$$

$\mathbf{z}_E$ denotes the vector of elastic coordinates, which is a single-row combination of the single elastic coordinates $\mathbf{z}_{Ei}$ of the base functions,

$$\mathbf{z}_E(t) = (\mathbf{z}_{E1}, \mathbf{z}_{E2}, ...)^T, \tag{3.10}$$

and $\mathbf{\Psi}$ is the Jacobian matrix of the elastic states,

$$\mathbf{\Psi} = \frac{\partial \mathbf{r}_{BV}}{\partial \mathbf{z}_E} d\mathbf{z}_E. \tag{3.11}$$

In general, a nonlinear dependency exists between the position $\mathbf{r}_{BV}$ of dV on the body and the elastic deformation. This would require that the modal coefficient matrices had to be computed for every new set of elastic coordinates; but with the presumptions given above, the equations of motion of the elastic deformation can be linearized. The modal coefficient matrices then become state-independent and can be computed prior to the integration of the equations of motion.

With the separation of nonlinear overall motion and small elastic deformation, the absolute position $\mathbf{r}_{IV}$ of a volume element dV was given by Equation 3.6. Under consideration of the definition of the Jacobian Ψ with its correlations

$$\begin{aligned} \mathbf{r}_{BV} &= \mathbf{\Psi} \mathbf{z}_E + \mathbf{r}_{BV(0)}, \\ \dot{\mathbf{r}}_{BV} &= \mathbf{\Psi} \dot{\mathbf{z}}_E, \\ \ddot{\mathbf{r}}_{BV} &= \mathbf{\Psi} \ddot{\mathbf{z}}_E \end{aligned} \tag{3.12}$$

the kinematic relations of the position vector $\mathbf{r}_{IV}$ from the inertial reference frame to the control element dV, the element's absolute transitional velocity $\mathbf{v}_V$ and its absolute transitional acceleration $\mathbf{a}_V$ can be written as

$$\begin{aligned} \mathbf{r}_{IV} &= \mathbf{r}_{IB} + \mathbf{r}_{BV}, \\ \mathbf{v}_{IV} &= \mathbf{v}_{IB} - \tilde{\mathbf{r}}_{BV} \omega_{IB} + \dot{\mathbf{r}}_{BV} \\ &= (\mathbf{E}, -\tilde{\mathbf{r}}_{BV}, \mathbf{\Psi}) \begin{bmatrix} \mathbf{v}_{IB} \\ \omega_{IB} \\ \ddot{\mathbf{z}}_E \end{bmatrix} \\ \mathbf{a}_{IV} &= \mathbf{a}_{IB} - \tilde{\mathbf{r}}_{BV} \alpha_{IB} + \tilde{\omega}_{IB} \tilde{\omega}_{IB} \mathbf{r}_{BV} + 2 \times \tilde{\omega}_{IB} \dot{\mathbf{r}}_{BV} \\ &= (\mathbf{E}, -\tilde{\mathbf{r}}_{BV}, \mathbf{\Psi}) \begin{bmatrix} \mathbf{a}_{IB} \\ \alpha_{IB} \\ \ddot{\mathbf{z}}_E \end{bmatrix} (\tilde{\omega}_{IB} \tilde{\omega}_{IB} \mathbf{r}_{BV} + 2 \times \tilde{\omega}_{IB} \dot{\mathbf{r}}_{BV}). \end{aligned} \tag{3.13}$$

3.5.2 Modeling Flexible Bodies in SIMPACK

SIMPACK has been designed as an open system which accepts various kinds of inputs from external standard software products. Interfaces have been established which allow links and transfers of various depth to and from other tools.

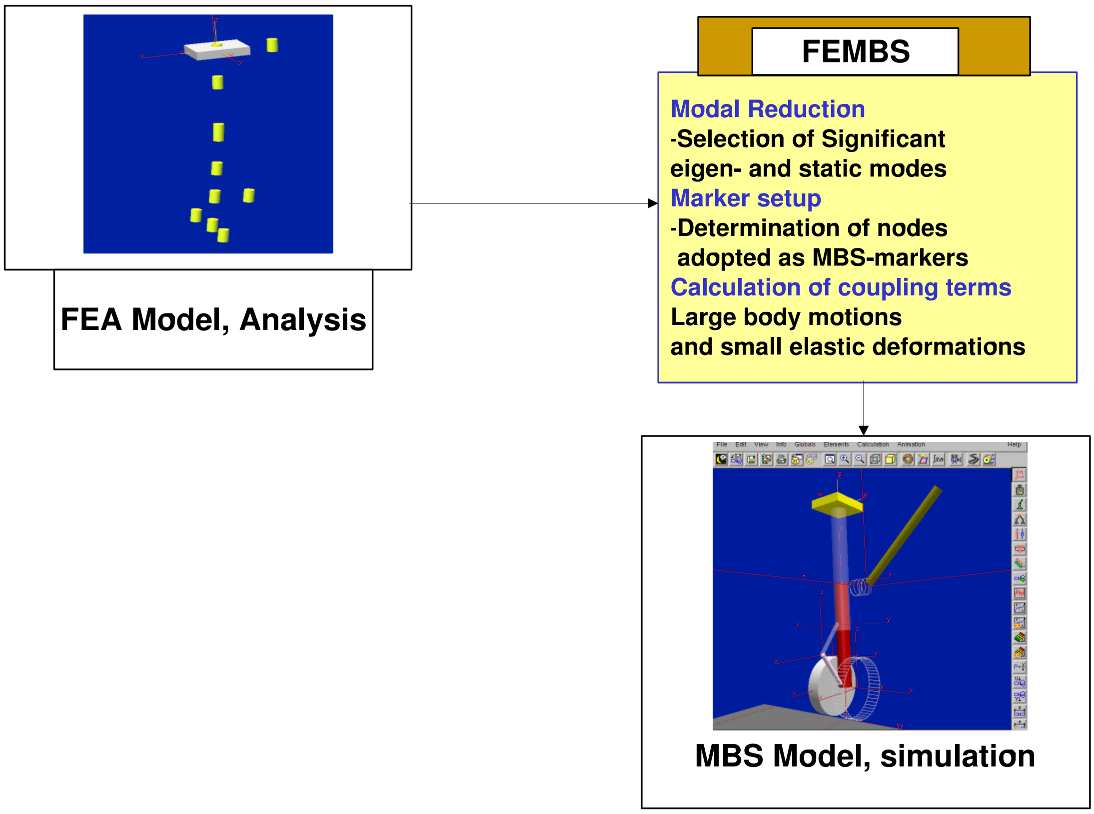


Figure 3.5: How FEMBS work in SIMPACK [72]

Two major options are available to analyze flexible mechanical systems: the finite element method (FEM) and the multi-body system approach. A simulation based on finite element models is, despite of the labor for setting up the model data, straightforward, the corresponding codes are well developed, and include linear and nonlinear theory of elasticity. Unfortunately, dynamic analysis with FEM-codes is very time consuming. In many applications one is confronted with system models, in which the deformations of the flexible bodies are small but superimposed of a large reference or 'rigid body motion'. In multi-body systems one exploits this fact to reduce the computational burden for such applications by linearizing the equations of motion assuming small deflections. Using relative variables to represent the reference motion and applying *O(N)*-formalisms [62]. In this work multi-body system approach has been preferred.

3.5.3 FEA Interface

Elastic bodies are transferred into SIMPACK using the modal approach. An elastic body is set up in a FEA tool and subject to an eigen value/eigen vector analysis. Mode shapes and nodes are transferred into the MBS model. The resulting deformation is a linear superposition of the mode shapes, see Figure 3.4.

The spacial motion of an elastic body is divided into a global motion, characterized by the movements of the body reference frame, and its elastic deformation which is expressed by the displacements of all (infinite) body points in relation to the body reference frame, see Figure 3.5. The global motion equals the rigid body motion of a classical rigid MBS body. The location and time dependent body deformation vector $u(r,t)$ is split by a separation function often referred to as Ritz approach into a location dependent displacement matrix $F(r)$ and the corresponding time dependent so-called elastic states $q(t)$. Each element of the vector q represents the influence of one eigen mode on the total response. The displacement matrix consists of mode shapes of eigen value and static load analysis. The eigen- and static modes as well as the stiffness matrix are computed in FEA; additionally, geometric stiffening effects, e.g. due to centrifugal forces, can be included.

Depending on the application often a relatively small number of low frequency modes are sufficient to represent, e.g., a static bending shape of an elastic body with sufficient accuracy. During the transfer to SIMPACK, the user is enabled to select only those modes which are necessary to describe the body flexibility for the individual load case. Thus, the full FEA model of system is replaced by a relatively small set of linear equations. The interface has been implemented for the FEA codes Nastran, ANSYS, ABAQUS and MARC [40, 72].

Chapter 4

Landing Gear Model

4.1 Introduction

The model of landing gear used for this work is a generic landing gear of a regional aircraft from Embraer. The rigid landing gear was prepared in SIMPACK as described in the Section 4.2. A multi-body model of a landing gear is prepared using various body elements which are connected to each other by means of joints. There may be force elements at the joints or between two body elements which are explained in Section 4.2.2. A rigid model, though sufficient for the dynamic load calculations, can not simulate the deformations that landing gear parts might undergo. To study these effects it was important to model the flexible landing gear. This also helped to detect the landing gear vibrations, namely gear walk and shimmy. It was further enhanced by means of control algorithm to study the brake and gear interaction. The various control algorithm will be explained in Chapter 5. Therefore a flexible model of landing gear was prepared in Nastran for modal analysis. As SIMPACK is designed as an open system it accepts inputs from various sources. The flexible model was transfered to SIMPACK using an interface called as FEMBS, please check the details in Section 3.5.3.

4.2 Modeling of the Rigid Landing Gear System

Figure 4.1 shows the schematic of a nose landing gear as a multi-body system [38]. In SIMPACK this multi-body system is represented by rigid body elements such as main-fitting, the shock tube, and two or four wheels, respectively. The shock absorbers (oleo) are located between shock tube and main fitting. All landing gears have one translational degree of freedom for the shock absorber and one rotational degree of freedom for each wheel.

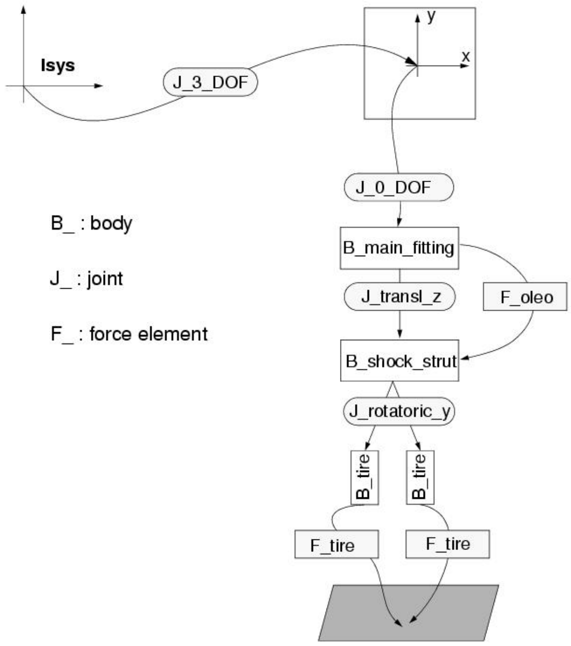


Figure 4.1: Schematic of a landing gear [38]

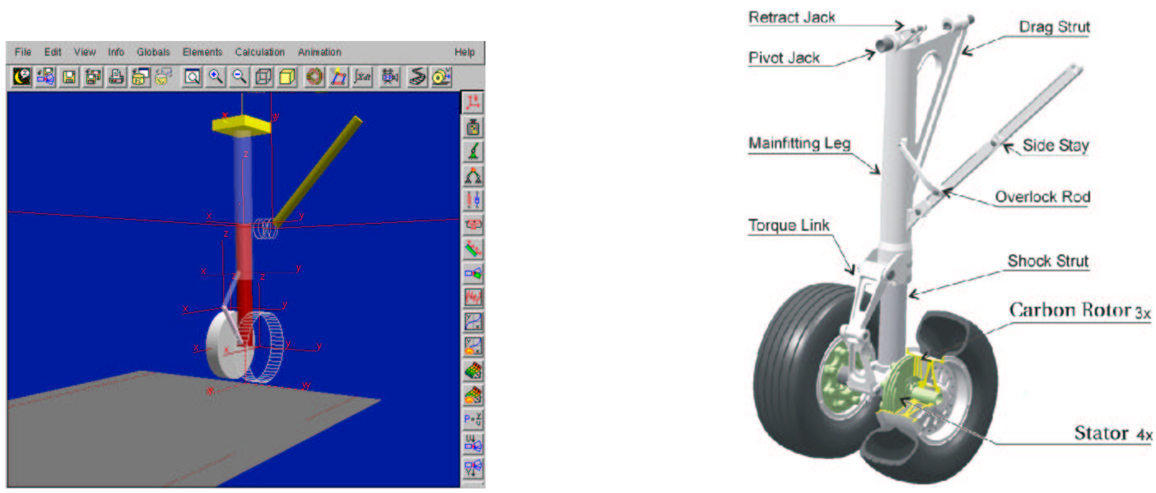


Figure 4.2: Left: screen-shot of a SIMPACK GUI, Right: CAD model of the landing gear

The main landing gears include an additional bogie attached to the shock tube with a rotational degree of freedom along the y-axis with 4 wheels attached to it. To model landing gears of large aircraft such as A380 main landing gear which has 6 wheels, a bogie, and a pitch trimmer in addition can be more complex. To model the system successfully one needs to define proper force elements to simulate the behavior of the whole system. SIMPACK has an in-built library of many force elements and it is also possible to write the so called user-routines which gives additional freedom to user to model different systems.

	Main Fitting	Shock strut	Tire and Brake System
Outer Diameter	9.4 cm	8.0 cm	70.0 cm
Inner Diameter	8.6 cm	7.0 cm	–
Length	1.469 m	1.24 m	17 cm
Weight	122 Kg	53 Kg	180 Kg per tire

Table 4.1: Mass and Geometrical Data Used for Modeling of the Main Landing Gear

	Mass	I_x	I_y	I_z
Main Fitting	122 Kg	**†	**	**
Shock Strut	53 Kg	**	**	**
Axle	70 Kg	1.74	1.744	6.285
Tire	50 Kg	11.86	11.86	6.86

Table 4.2: Mass and Moment of Inertia Used for Modeling of the Main Landing Gear.

4.2.1 Data for Modeling Purposes

The Embraer aircraft modeled in this work has a hydraulically actuated tricycle landing gear comprising the main landing gear (MLG) and the nose landing gear (NLG). The main landing gear consists of a leg, shock strut and wheel with wheel brake. The MLG are located in the fuselage center section nacelle fairings and retract in an inward direction to be closed off by the MLG doors. The nose landing gear consists of a leg, shock-strut and double wheels with NLG steering and retracts in forward direction into the NLG actuator.

Data provided in Table 4.1 and Table 4.2 is used to model the main landing gear of the Embraer regional aircraft, please see Figure 4.2. This section gives dimensions and the other details of various parts.

†Nastran Model Input

4.2.2 Force Elements

Force elements apply external or internal forces and torques in the system. They may depend upon the state of the system, eg. the distance between two points, and upon time. Force elements do not affect the degrees of freedom of the system, but may introduce additional states, or boundary conditions, to the differential equation system of the MBS model. SIMPACK has an in-built library of force elements to enhance the model, it also allows to add the so-called "user routines" where use can define new element. The force elements describing the landing gear characteristics have been modeled in detail for this work by means of so-called user-routines in SIMPACK. While the equations of the physical phenomena as such are valid independently from the exact aircraft type and can be taken from standard textbooks [12, 52], the parameters for the force elements are usually proprietary.

Hydro-pneumatic Oleo

For transport aircraft the main task of vertical energy dissipation is almost exclusively taken over by an oleo-pneumatic shock-strut. This device combines a gas spring with oil and additional friction damping [41]. Damping force is provided by oil flow forced through an orifice by vertical strut motion. Often the oil flow is controlled by means of metering pin. The gas spring is represented by a law of polytropic expansion [49] as

$$F_f = F_0 \left(1 - \left(\frac{s}{s_m}\right)\right)^{-n \cdot c_k} \tag{4.1}$$

with spring force F_f, pre-stress force F_0, oleo stroke s, oleo gas length s_m, polytropic coefficient n $(1 \leq n \leq k)$, and a correction factor c_k. The pre-stress force F_0 can be calculated from the initial pressure in the fully extended oleo. The correction factor c_k, typically between 0.9 and 1.1, allows the adjustment of the curve to measured data. The minimum and maximum stroke limits are modeled by stiff springs. The properties of the passive damper are determined by the laws describing the flow of a viscous fluid, e.g. oil, through an orifice.

$$F_d = \pm|\dot{s}| \cdot d \cdot \dot{s}^2 \tag{4.2}$$

with oleo stroke velocity $\dot{s}$, oleo damping force F_d, and damping coefficient d. An input function for a spring and damper is as shown in Figure 4.4.

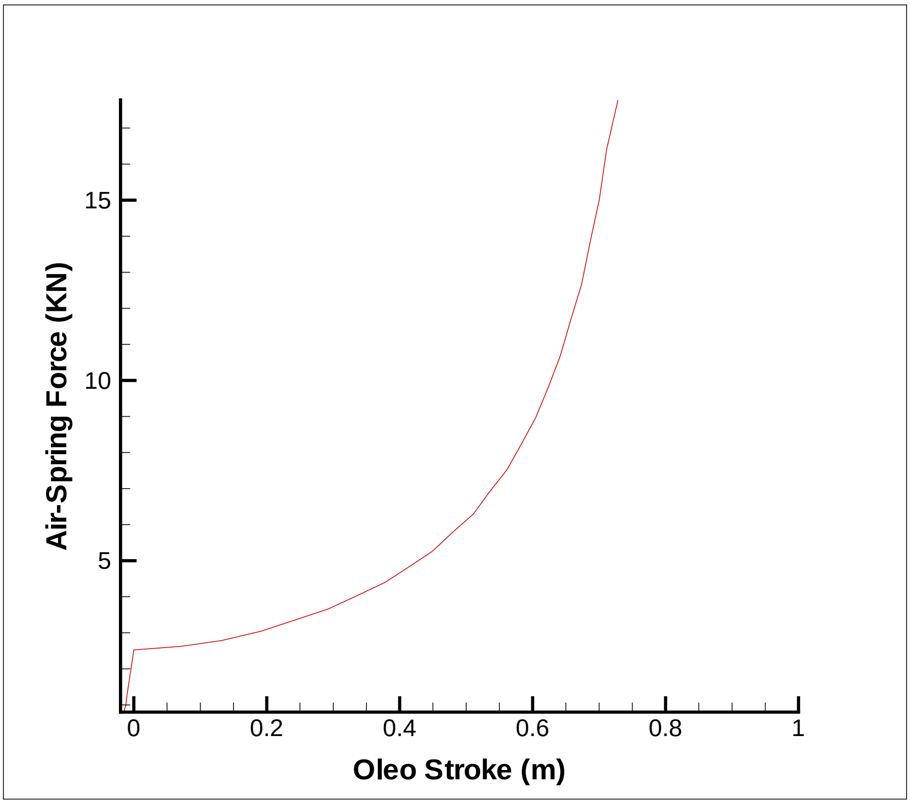


Figure 4.3: Input function curve for oleo-spring

Tire Force

The tire model developed at the DLR takes vertical, longitudinal, and lateral effects into account. The tire connects the wheel to the runway when the aircraft is on the ground. The simulation force element measures the height of the wheel axis with respect to the excitation. This rolling radius r_r is subtracted from the nominal tire radius, r_{nom}, to determine the tire deflection d_z

$$d_z = r_{nom} - r_r \tag{4.3}$$

The wheel is modeled as a separate body with a rotational degree of freedom. The longitudinal and lateral motion of the body with respect to the runway is used to calculate tire slip and torque on the wheel. The vertical force F_z is calculated first. It is a function of the tire deflection d_z. Using a third-order polynomial we find

$$F_z = c_1 \cdot d_z + c_2 \cdot {d_z}^2 + c_3 \cdot {d_z}^3 \tag{4.4}$$

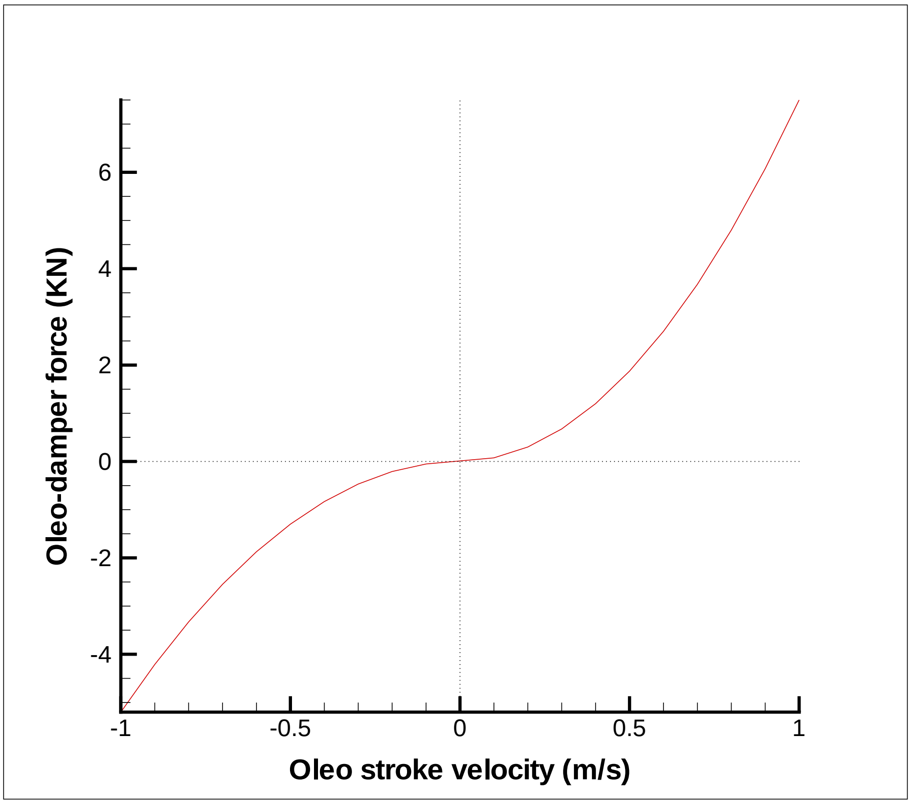


Figure 4.4: Input function curve for oleo-damper

where c_1, c_2, and c_3 are selected to match measured tire data. A linear spring can be simulated by setting c_2 and c_3 equal to zero and providing the spring coefficient in c_1.

For longitudinal forces the slip calculated in the main tire element is used. It is defined as the ratio between the horizontal velocity of the wheel contact point and the axle forward velocity,

$$Slip_{longitudinal} = \frac{v_x - r_r \cdot \Omega}{v_x} \tag{4.5}$$

where Ω denotes wheel spin and v_x the wheel axle forward velocity. The friction coefficient μ_{rw} of the runway is a function of slip. An approximation of the functional relation between μ_{rw} and slip is displayed in Figure 4.5

Typical values for μ_1 and μ_2 range from 0.4 to 0.9 for dry runways, depending on the runway type. The friction coefficient μ_{rw} is needed to calculate the longitudinal tire force F_x which is a function of the vertical tire force F_z and μ_{rw}

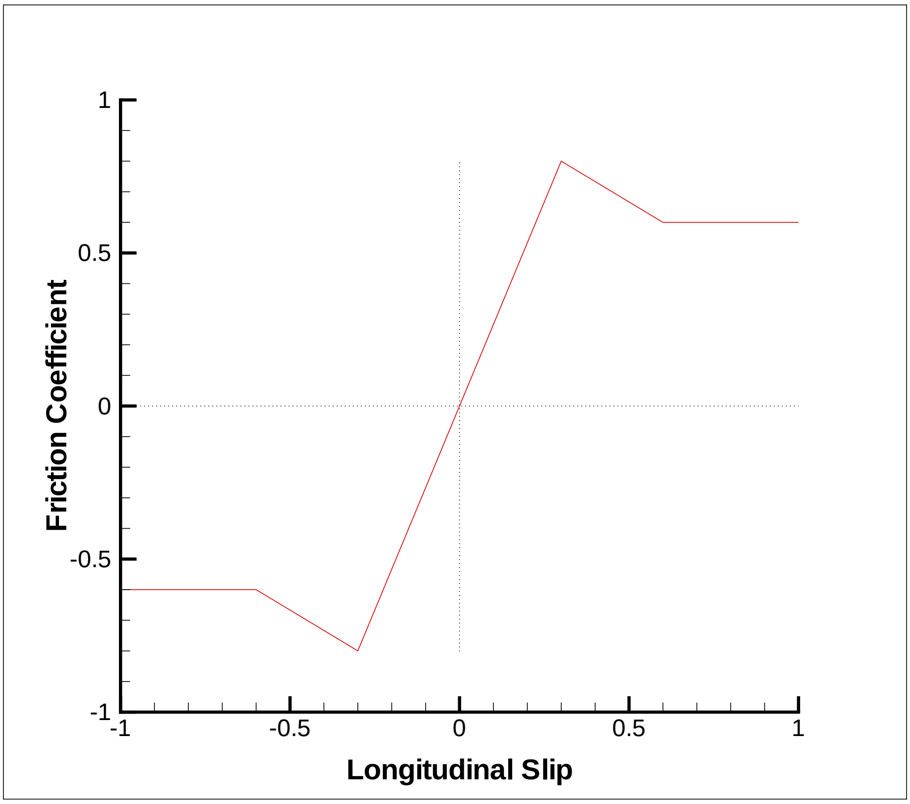


Figure 4.5: Functional relation between friction coefficient and slip

$$F_x = \mu_{rw} \cdot F_z \tag{4.6}$$

The resulting torque T_y on the wheel is calculated using the effective rolling radius $r_{r,eff}$ which can be set to a constant value or, if desired, can be calculated during the simulation using the equation

$$r_{r,eff} = r_{nom} - \frac{d_z}{3} \tag{4.7}$$

The torque T_y is then

$$T_y = r_{r,eff} \cdot F_x \tag{4.8}$$

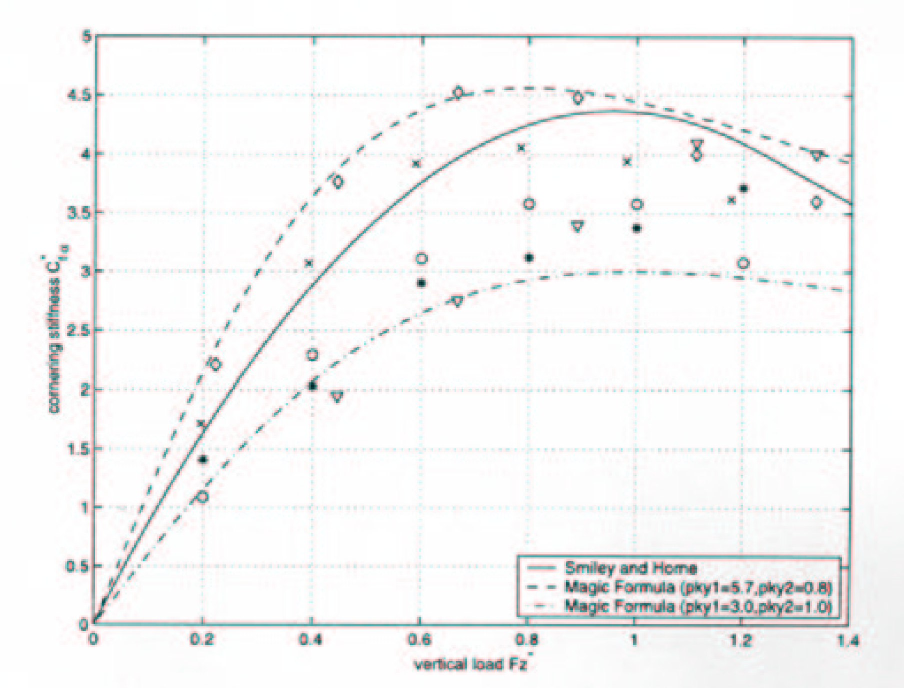


Figure 4.6: Measurement and empirical formulas for cornering stiffness, aircraft tires

Tire Cornering Stiffness

For the tire behavior, a number of different parameters are important. Next to the vertical stiffness the cornering stiffness is of special importance for lateral loads. The cornering stiffness $C_\alpha(N/rad)$ is generally defined as

$$C_\alpha = \frac{\partial F}{\partial \alpha} |_{\alpha=0} \tag{4.9}$$

The cornering stiffness is a function of vertical load. Values are not readily available from the tire manufacturers but can be deducted from experience and related work. Figure 4.6 gives a typical normalized curve of vertical load vs.cornering stiffness from which it appears that $C_\alpha = 3.0..4.5 \cdot F_Z$ is a reasonable assumption.

In flight mechanics and from model identification experience the following relation is known: $F_y = \mu \cdot F_z \cdot \alpha$. Using the above equation for C_α this can be re-written as $C_\alpha = \mu \cdot F_z$. A realistic value of $\mu = 0.05(deg) = 2.86(rad)$ has been chosen in former simulations which is at the lower end of the values given by Basselink [3].

Another approach is taken by Smiley and Horne from measured data who give the following equation based on tire width w, tire pressure p, rated tire pressure p_r, tire deflection δ and unloaded tire radius R_0:

$$\begin{aligned} C_{f\alpha} &= 57 \cdot {\omega_t}^2 (p + 0.44 p_r) \left(1.2 \frac{\partial}{2R_0} - 8.8 \left(\frac{\partial}{2R_0} \right)^2 \right), \quad \frac{\partial}{2R_0} \leq 0 \\ C_{f\alpha} &= 57 \cdot {\omega_t}^2 (p + 0.44 p_r) \left(0.0674 - 0.34 \left(\frac{\partial}{2R_0} \right) \right), \quad \frac{\partial}{2R_0} \geq 0 \end{aligned} \tag{4.10}$$

Since it is so difficult to exactly determine C_α its influence should be checked by a parameter variation during the analysis.

4.2.3 Modeling Flexible Main Landing Gear for Embraer Regional Aircraft

A valid landing gear simulation is one having the same dynamic response to brake torque as the actual gear. This means that the simulated gear must be designed to have the same equation of motion in its walk mode under the action of speed-dependent braking friction [18]. The traditional way to simulate the gear has been to use alternate structure, a dynamo-meter fixture such that one of its fundamental modes duplicates the dynamic characteristics of the gear walk mode of interest. In this work, the flexible multi-body dynamics methods are used for the simulation of such an unstable and complex phenomenon during aircraft ground maneuvers to detect friction-induced vibrations in aircraft landing gear.

The landing gear model is prepared in Nastran as a beam model with the help of data exchanged with the industry partner Liebherr for a newly developed regional aircraft. The landing gear is modeled for different strokes and the results of the modal analysis are compared to the model received from the Liebherr and fine tuned to get similar eigen shapes and eigen frequencies. To create the model in Nastran following data given in Table C.1 is used. The wheel axle is attached with the rotational degree of freedom around the y-axis at the end of beam number two. The wheels are represented by condensed masses. The results of the modal analysis for zero stroke are given in Table 4.4. The Nastran model is then imported in SIMPACK as explained in the Section 3.5.3.

There have been two different models of flexible landing gear prepared. After the touch down it is safe assume that the landing gear remains at a level as shown in Figure 4.8. The shock strut travel (stroke value) inside the main-fitting can be

Parameter	**Beam1**	**Beam2**
$D_{1,2}$	0.15 m	0.13 m
$L_{1,2}$	1.469 m	1.188 m
$t_{1,2}$	0.005 m	0.004 m
$E_{1,2}$	2.1E+11	2.1E+11
$\rho_{1,2}$	7895 Kg/m^3	7895 Kg/m^3

Table 4.3: Data used for the Two Beam Landing Gear Model in Nastran.

Mode Number	Eigen Frequency, Hz	Eigen Shape
1	10.10	Torsion, Lateral
2	11.22	Fore-aft, Side stay tangential
3	13.25	Torsion, Side stay radial
4	45.69	Second Lateral
5	62.31	Vertical mode of the wheels

Table 4.4: Results of modal analysis in Nastran.

assumed to remain constant safely. The first model prepared in Nastran is based on the assumption that the stroke value is 0.478m which is obtained from the flight-test data. The second model prepared is based on the assumption that the stroke is variable, ie. the shock-strut has a translational degree of freedom in Z direction. The modal analysis was done on both these models. The eigen frequencies and mode shapes after the analysis are shown in Figure 4.9 and 4.10.

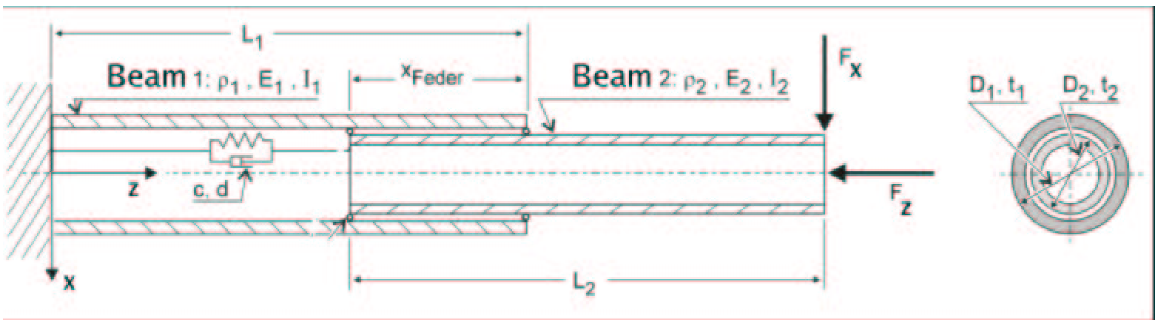


Figure 4.7: Beam model of the landing gear [42]

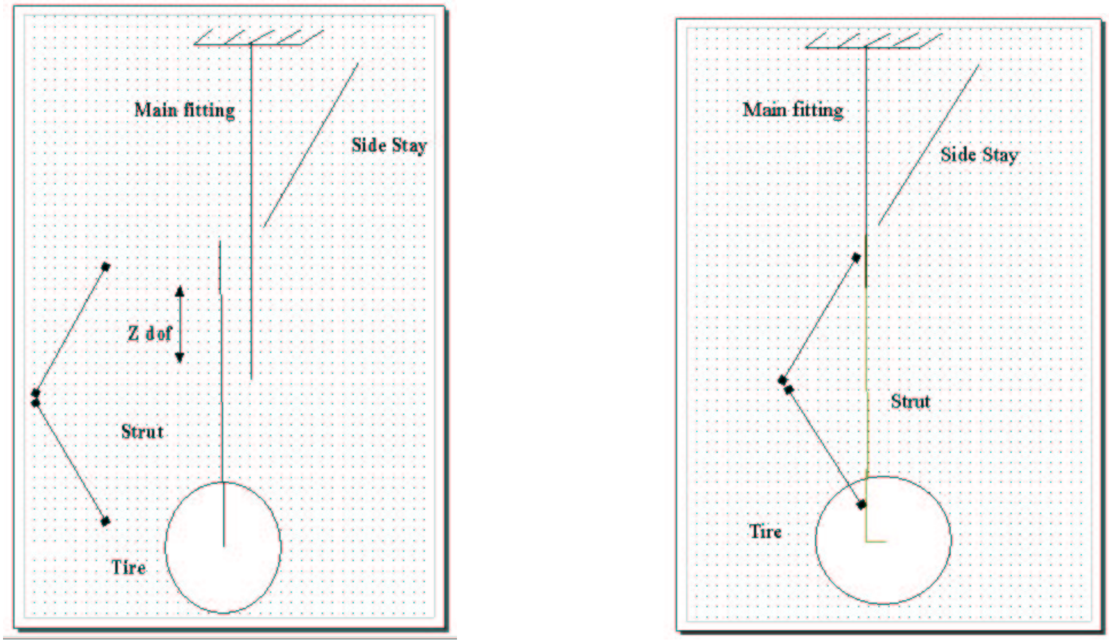
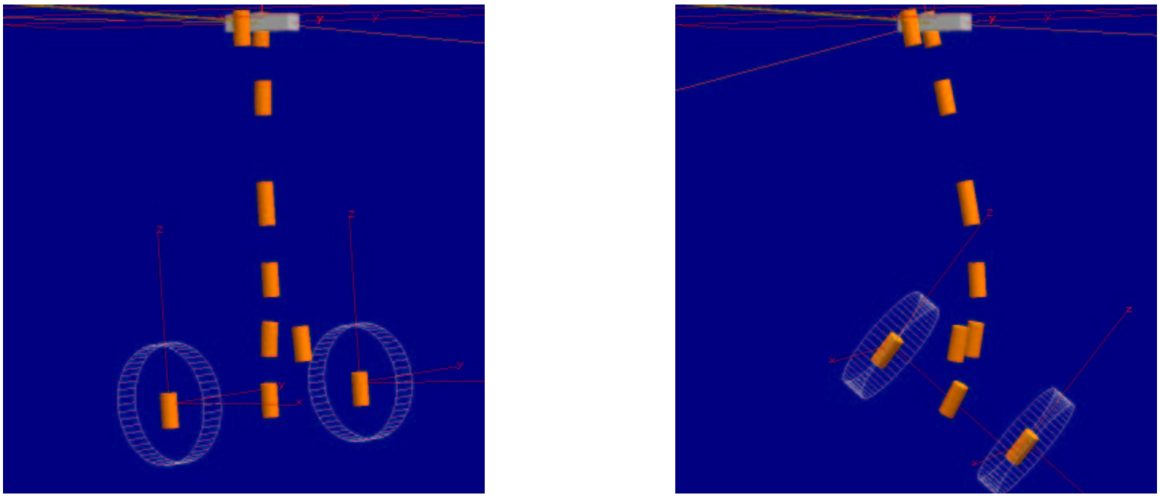


Figure 4.8: Modeling approaches of flexible landing gear in Nastran

4.2.4 Excitation and Free-play

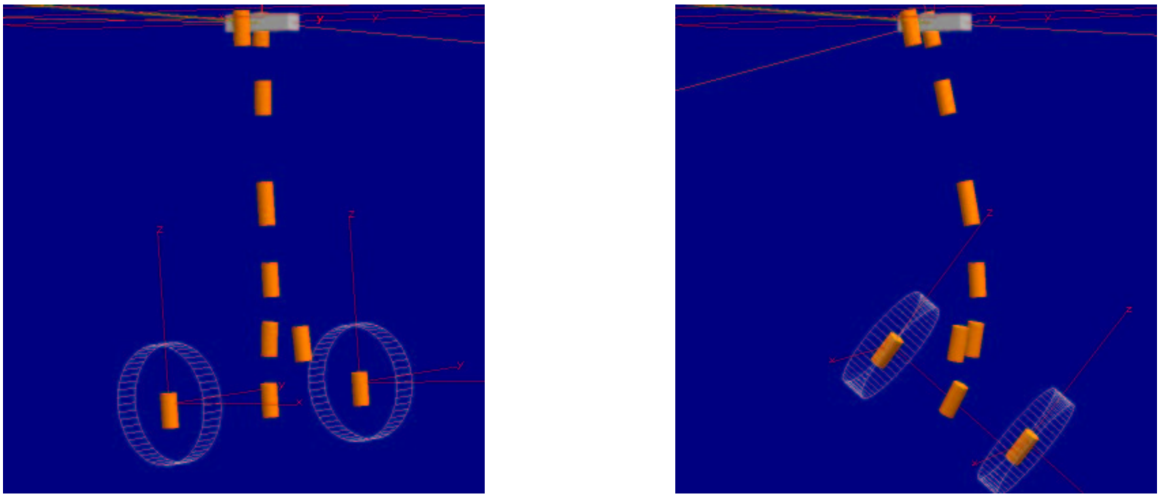
For the non-linear time simulations, the landing gear is excited by means of a load and a torque peak on the axle. The load start at time t_0, reach their maximum value at $t_0 + 0.001s$ and are zero again at $t_0 + 0.002s$. The value of the load in y direction is $20000N$, the torque around z is $20Nm$ as shown in Figure 4.11. The values have been arbitrarily chosen but are of the same magnitude as the forces and moments which occur at a hard landing [39].

In case of parts of a structure sliding or moving with respect to each other free-play exists. Landing gear is a classical case of a structure having lot of free-play in its members moving with respect to each other. Typical examples are the torque link hinges, apex joint, upper and lower bearing between main fitting and sliding member, various hinges in the side stay assembly and pintle lugs. Due to these parts a fair amount of free-play can be expected at the wheel axle center, typically in the order of millimeters in lateral and fore-aft direction and less than one degree in yaw [3]. Due to changes in torque link geometry and overlap, it can be expected that the amount of free-play at the wheel axle center in fore and aft, lateral and yaw direction will be a function of the shock absorber closure. It can be represented fairly accurately by lumping the lateral free-play in side stay and yaw free-play as a translational degree



(1) **(2)**

Figure 4.9: Eigen-modes of landing gear; Left: 10.10 Hz torsion-lateral, Right: 11.22 Hz fore-aft and side stay tangential



(3) **(4)**

Figure 4.10: Eigen-modes of Landing Gear; Left: 13.25 Hz torsion and side stay radial, Right: 45.68 Hz second lateral

of freedom at the apex joint. For the simulation, values for the free-play are taken from landing gear design [3, 39]. It is taken into account as two additional rotational degrees of freedom in the joints between piston and cylinder.

Free-play is modeled as non-linear spring, see Figure 4.12. Some deflection is possible before a force develops and if the amplitude remains within the free-play band the spring force will be zero. Grossmann [26] suggests two different formulae to determine an equivalent linear stiffness c_{eq} as a function of amplitude outside the free-play band ($a_q > a_{fp}$).

$$c_{eq} = c \cdot \left(1 - \frac{a_{fp}}{a_m}\right) \tag{4.11}$$

$$c_{eq} = c \cdot \left(1 - \left(\frac{a_{fp}}{a_m}\right)^2\right) \tag{4.12}$$

with a_m the amplitude of the motion and a_{fp} half of the free-play. Obviously the stiffness has become a function of the amplitude of the motion and will increase with the amplitude of the motion, see 4.13. Equation 4.11 gives better correspondence with the non-linear simulations than Equation 4.12 [3], also suggest that the equivalent stiffness may even be lower than estimated

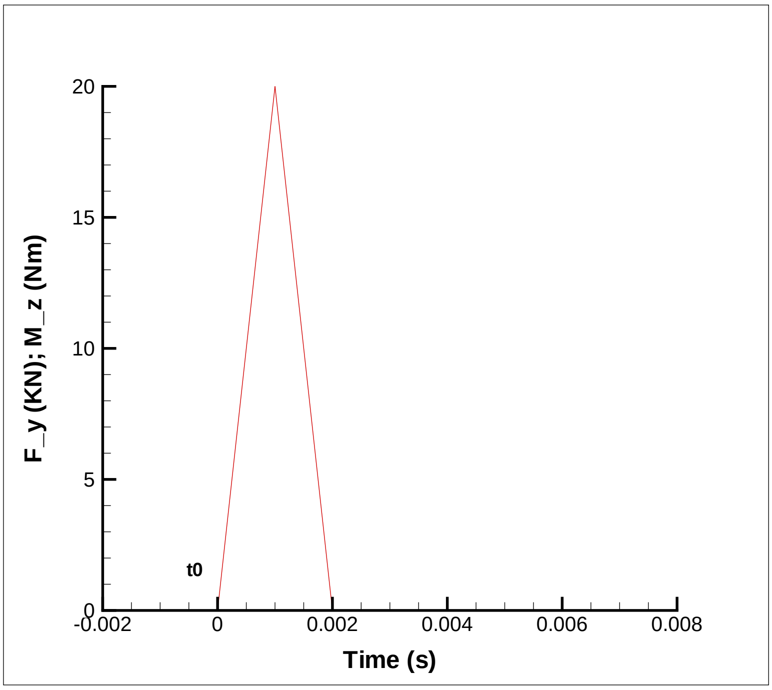


Figure 4.11: Excitation load and moment acting at axle [39]

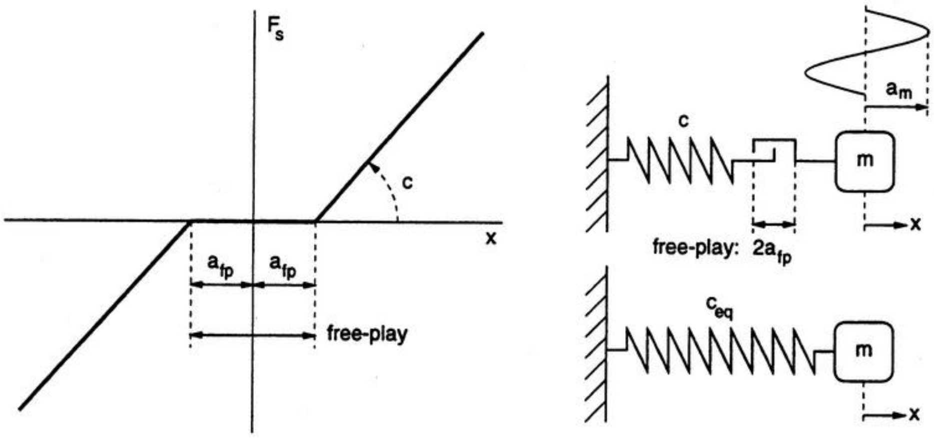


Figure 4.12: Representation of free-play [3]

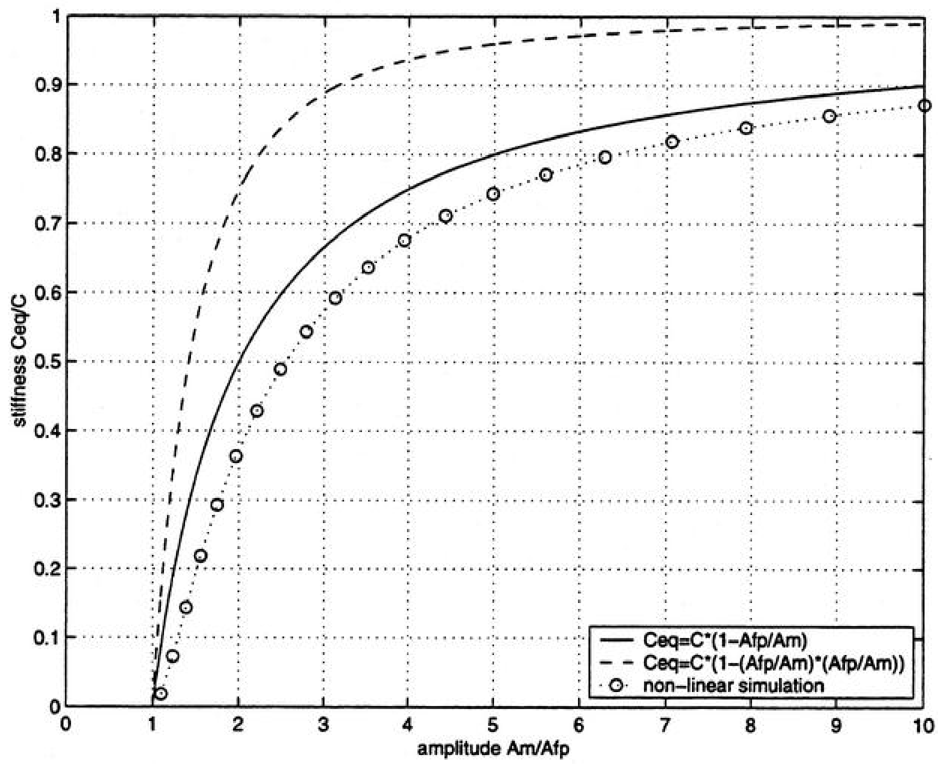


Figure 4.13: Effective stiffness as a function of motion amplitude for a system with free-play [3]

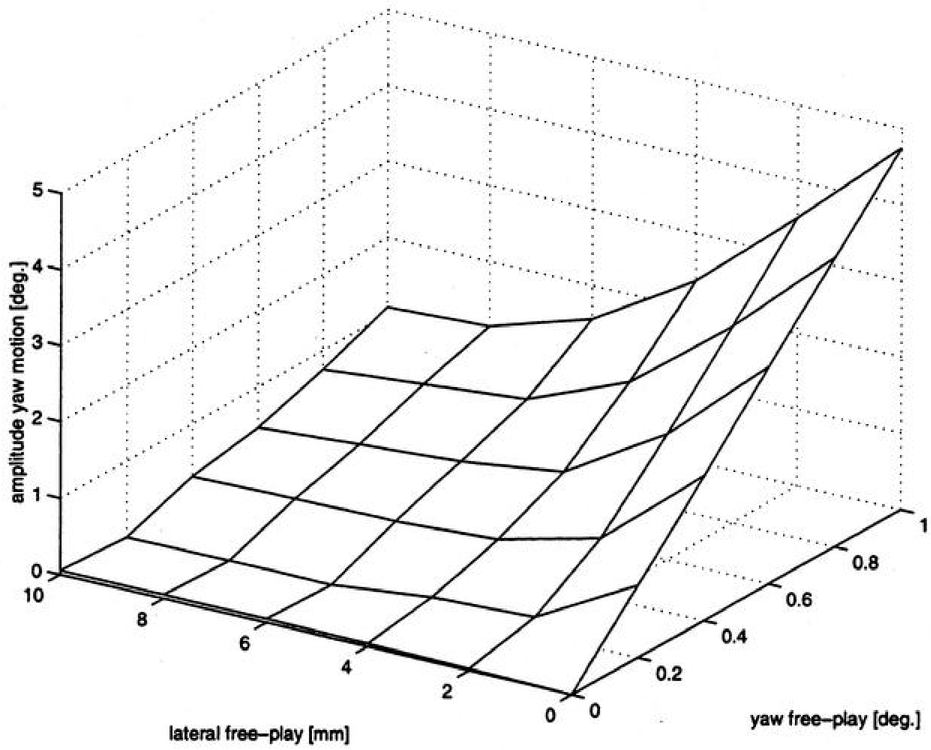


Figure 4.14: Limit cycle amplitude as a function of yaw and lateral free-play [3]

Chapter 5

Modeling of Brake Algorithm

5.1 Braking : Algorithm and Dynamics

Stopping of the aircraft being their primary task, brakes are also used to control speed while taxiing, to steer the aircraft through differential action, and to hold the aircraft stationary when parked and during engine run-up. They are generally fixed to the main gears only and add substantial weight to them. Most airplanes use disk brakes in conjunction with an advanced anti-skid control system. In this work two different braking algorithms have been implemented and tested for different simulation cases. *Dynamic Braking* - which is explained in details in the earlier work [34] is negative torque acting at the wheel axle and the *Anti-skid Braking System* where slip is optimized in order to avoid locking. This braking algorithm based on slip optimization principle was further refined by including hydraulic dynamics and is called as Modified Anti-Skid Braking Algorithm.

5.1.1 Dynamic Braking; Braking with Negative Torque

Dynamic braking is the method which is normally used for flight-test cases and is a crude art of braking. Consider the forces and torques on one of the landing gear wheels as shown in the Figure 5.1, where F_n is the normal force on the tire, V_a is the forward velocity, and T_b is the braking torque. If we write down the general form of force equations they will look like this

$$\begin{aligned} \mathbf{M_y} &= \mathbf{F_l} \times \mathbf{r_r} - \mu_b \times \mathbf{F_n} \times \mathbf{r_r} \\ &= \mathbf{T_b} \end{aligned} \tag{5.1}$$

where μ_b is the braking force coefficient, $\mathbf{F_l}$ is force in forward direction, $\mathbf{M_y}$ is the moment about y-axis, ζ is the yaw angle, $\mathbf{r_r}$ is the deformed tire radius. According to JAR standards for dynamic braking the following curve shown in Figure 5.2 is used as a torque acting between the axle and the inertial system. As a rough guideline we have used Equation 5.2 to calculate these values.

$$2 \times \mathbf{T}_b = \begin{cases} 0.0 & \text{if } t \leq 0, \\ 0.6 \times F_n & \text{if } t = 0.4\text{s}, \\ 0.8 \times F_n & \text{if } t \geq 0.9\text{s}. \end{cases} \tag{5.2}$$

5.1.2 Anti-lock Braking System

The coefficient of friction between the tire and the runway (or road-surface) depends mainly on the slip ratio, normal force, forward velocity, and runway conditions (damp, rain, ice, snow). The nature of this dependence is not well understood even after numerous experiments. If we assume that other factors are fixed the friction coefficient can be represented as a function of slip ratio [69, 34] and has a unique maximum, see Figure 5.3. During braking it is possible that the wheels get locked. This occurs when the applied braking torque exceeds the friction torque between the tire and the surface, reducing the wheel angular speed to zero, i.e. the slip ratio becomes equal to one. In such a case the airplane should be equipped with an Anti-lock Braking System which prevents wheel locking. In addition it should also try to maximize the friction coefficient between the tire and the runway surface, in order to minimize the stopping distance. Achieving a shorter stopping distance becomes critical on wet or icy runways and during Rejected Take-Offs (RTO). It may also be designed to enhance passenger comfort through reducing strut vibrations and improving tire wear through smoother braking, as secondary objectives [69].

For this purpose a simple and somewhat idealized anti-skid braking algorithm has been implemented at the DLR which works as follows: Figure 5.4 shows the schematic of an Anti-skid braking algorithm where the state-space model give the kinematic measurements (eg. Actual velocity, slip etc) to the braking algorithm. A sensor at the landing gear wheels measures the actual speed and angular velocity which is fed to the control system along with the desired velocity and the desired slip. The actual slip is determined. If the actual velocity is greater than the desired velocity and the actual slip is not equal to the desired slip then the anti-lock braking system is activated. There is a bang bang controller which brings ABS into action when the slip falls below the desired level and releases the brakes when the slip increases.

5.1.3 Role of Hydraulics

The results of the simulations based on the simple 'slip-optimized' braking showed that it is necessary to sophisticate the braking algorithm. This is done by means of

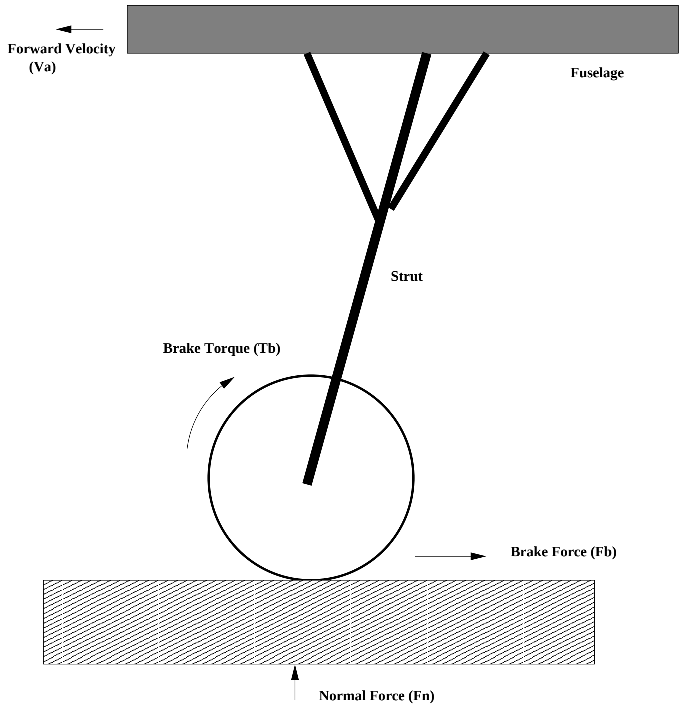


Figure 5.1: Schematic of a Wheel during Braking

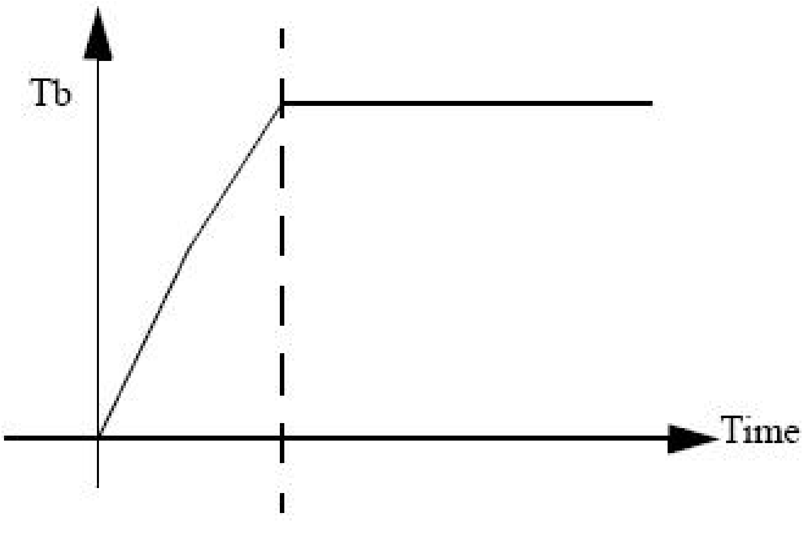


Figure 5.2: Typical dynamic braking torque curve.

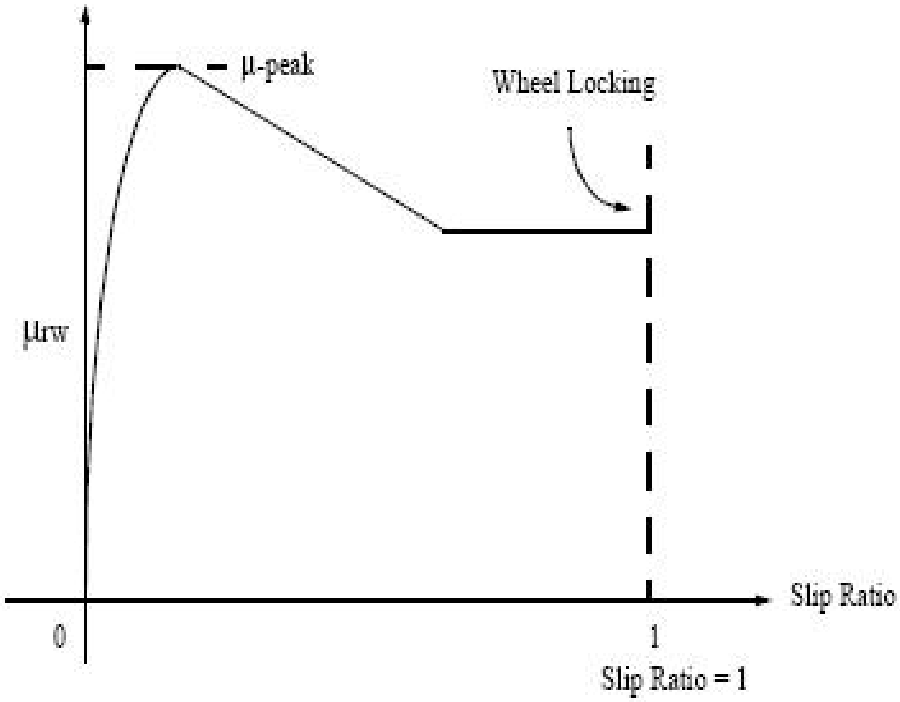


Figure 5.3: Friction coefficient as function of slip ratio

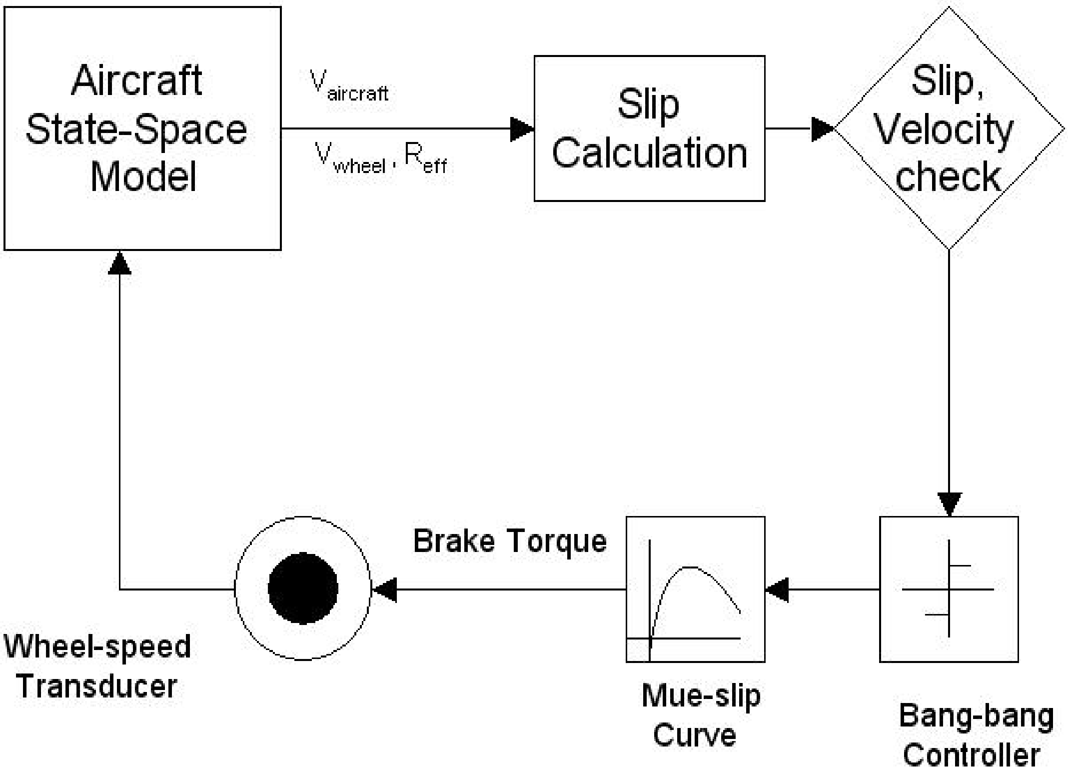


Figure 5.4: Anti-lock braking algorithm based on slip optimization

hydraulic lag and modeling 'real-life' like braking algorithm instead of simple braking based on 'slip-optimization' principle. The dynamics of the hydraulics is taken into the consideration as a part of the modified braking algorithm in order to study the effects of changes in the geometrical parameters such as thickness and length of the hydraulic lines. The effects of changes in the viscosity of the hydraulic fluid due to the temperature rise will be studied in the future work.

The pressure drop in the landing gear hydraulic lines is based on the assumption that the fluid flow state inside the pipe line will be either laminar or turbulent. A switch based on the Reynolds number is used in the calculation. The pressure drop equation is given as [46, 75]

$$\mathbf{P}_{in} - \mathbf{P}_{out} = \begin{cases} \frac{64}{R_e}\frac{L_{pipe}}{D_{pipe}}\frac{v^2}{2} & \text{if } R_e \leq 2000 \text{ - Laminar Flow,} \\ \frac{0.3164}{R_e^{1/4}}\frac{L_{pipe}}{D_{pipe}}\frac{v^2}{2} & \text{if } R_e \geq 2000 \text{ - Turbulent Flow.} \end{cases} \tag{5.3}$$

5.1.4 Modified Anti-lock Braking Algorithm using Hydraulic-dynamics

The block diagram of new brake control architecture is shown in Figure 5.5. It is explained in details in the simplified control diagram shown in Figure 5.6. The brake control module receives inputs from the aircraft system, which includes desired velocity profile for different ground maneuvers, optimal slip values based on the runway and atmospheric conditions (rainy, icy etc.) as well as feedback from the brake system, which includes actual velocity and slip values, also information about the malfunction of system or part of it. The pedal transducer input is also given to the brake control module which works on the principle of LVDT (Linear Variable Displacement Transducer). Pedal position is derived from LVDT's and inputed as an electrical command to the brake control module. On the ground, the brake control module provides outputs to the shut-off valves enabling pressure supply to the individual wheel brake control valves. Pressure valves command brake pressure proportional to the brake pedal position command. Pressure feedback signals from the pressure transducers is used to ensure close correlation between commanded and resulting brake pressure. Axle mounted wheel-speed transducer signals are processed to provide an anti-lock correction signal to the brake command signal to result in a brake pressure command that ensures maximum braking force and peak available wheel-runway slip velocity.

Initial brake pressure input to the Anti-skid valve which adjusts the brake pressure by means of optimizing the slip as explained in the Section 5.1.2. The brake pressure is then given as input to the brake system through hydraulic fuse. It uses hydraulic pipes to transmit this pressure. The losses are calculated as explained in Section 5.1.4. Some important parameters of brake system data are listed in Table 5.1. Skydrol is a common hydraulic fluid used in the brake system to transmit the brake pressure to the brake system. The properties of Skydrol are given in Table 5.2.

Parameter Name	**Value**
Mass of Brake System	60.96 Kg
Mass of Tire	73 Kg
Mass of Wheel	47.099 Kg
Brake MI	0.86 Kgm^2
Tire MI	10.26 Kgm^2
Wheel MI	2.09 Kgm^2
Brake Piston Area	1316.6 mm^2
Hydraulic Pipe Length	3.4 m
Hydraulic Pipe Diameter	9.52 mm

Table 5.1: Data of Brake System for One Wheel

Parameter Name	**Value**
Kinematic Viscosity	$10.8 - 11.6 \times 10^{-}2\ m^2/s$
Specific Gravity	$1.009 - 1.019 \times 10^3\ Kg/m^3$

Table 5.2: Properties of Skydrol

There are two different control elements involved in this control algorithm. Connection Element and Function Generator is a comparator and switch, available as standard SIMPACK filter Nr.143. This is used as a comparator to compare the actual values to the desired values of veriables such as velocity, slip etc. More information about this approximation method is given in literature about digital filters and digital control [22]. Discrete or Analog Filter by Transfer Function, control element Nr. 140 in SIMPACK, is an AD converter. This filter is used for the purpose of defining time lag which is essential and is part of the whole braking logic. It allows smooth braking, the value of lag depend on the sampling time and affect the vibrations in landing gear. Please refer to Appendix A and Appendix B for further details on how it works.

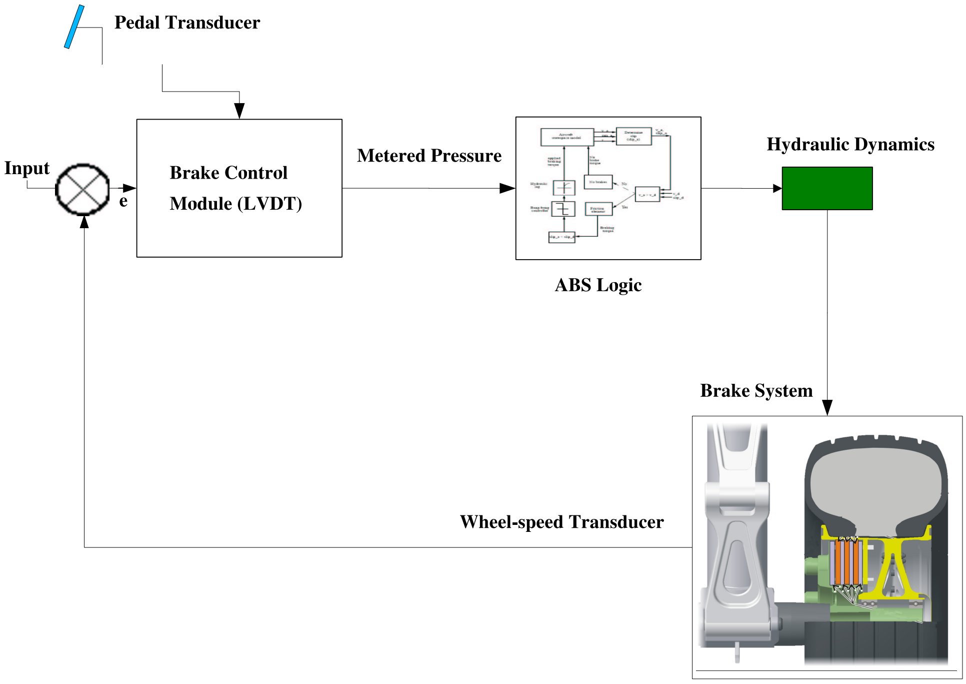


Figure 5.5: Block diagram of Brake Control system architecture for modified braking

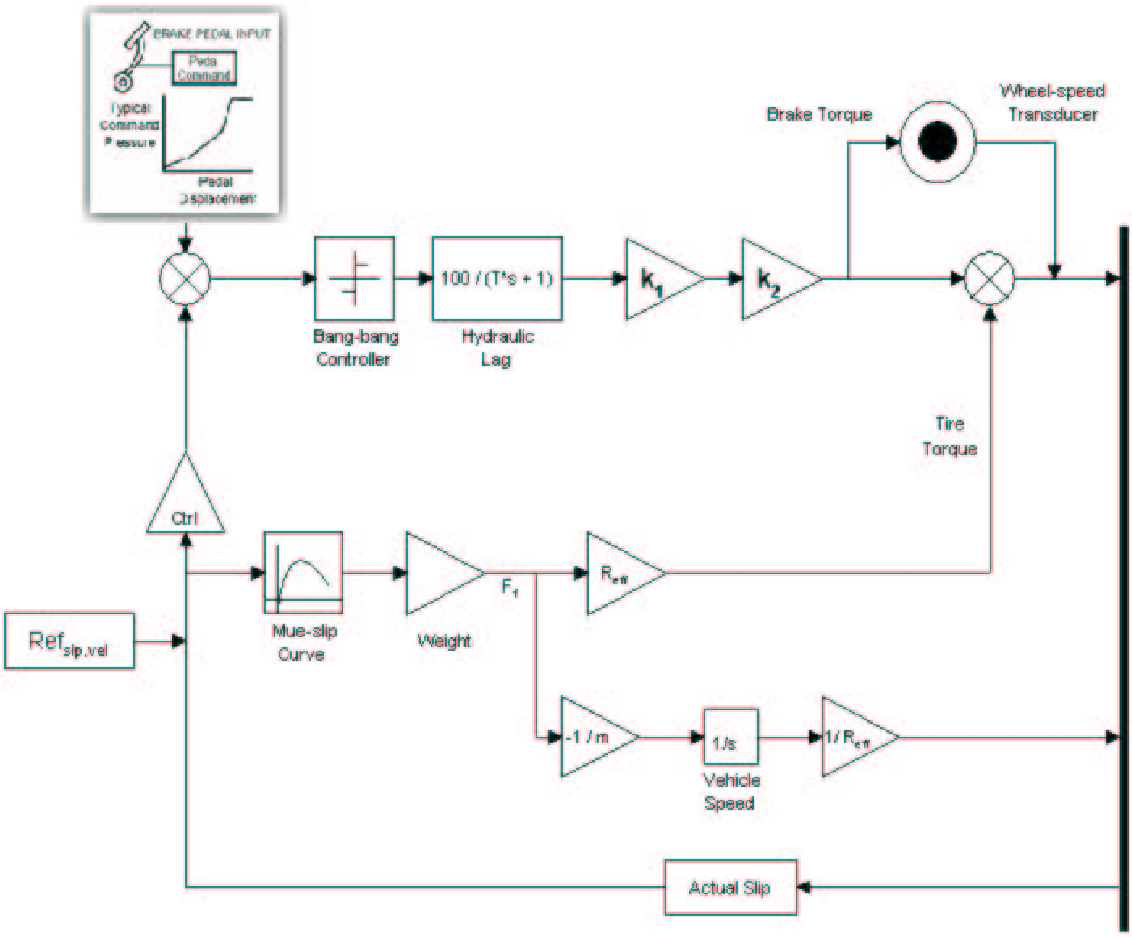


Figure 5.6: Brake control system logic for modified braking

Chapter 6

Simulation and Results

Engineers designing the landing gear system face the daunting task of tracking down and correcting vibration sources in it. In the last decade or so OEMs (Original Equipment Manufacturer) have cut down the time to deliver the aircraft. So they do not have the luxury of iteratively refining the design by means of experimental testing of the prototype. Dynamic simulation of the entire landing gear system is a faster and very accurate way, thanks to the CAE tools that are available today. It can also be used quickly for different aircraft system as the basic modeling tasks such as brake-algorithm and tire model are ready to use once finished. To understand the aircraft ground dynamics and to determine realistic ground loads a simulation of operational cases with an accurate model is thus necessary.

In this work, a flexible multi-body model of a landing gear has been prepared by means of state-of-the-art tools and has been evaluated by means of various important ground maneuvers. Different modeling tasks included development of a tire model with lateral dynamics to calculate the cornering forces during a curved run, braking system with an Anti-skid algorithm and its effect on the aircraft performance in terms of stopping distance and passenger comfort. The goal of the work was also to study landing gear and brake interaction and the related friction induced vibrations such as gear walk and shimmy.

6.1 Simulation Cases

Various simulation cases were designed in order to study the friction induced vibrations in landing gear and parameters that affect the longitudinal and lateral vibrations. The results presented in this work are for a two mass model main landing gear for the Embraer regional aircraft. The simulations were done to study the landing gear vibrations, the effect of critical parameters of hydraulic lines on the same, and various parameters affecting landing gear shimmy. For the simulation of rolling and braking maneuvers it is safe assume that the aircraft is on the ground. The initial simulation case were done, as explained in Section 4.2.3, with the flexible landing gear modeled using the first approach. For this case the landing gear was modeled assuming that the stroke remains constant, See Figure 4.8. After initial studies landing gear with variable stroke was also modeled.

6.1.1 Dynamic Braking and Braking with Slip Optimization

A good anti-skid algorithm should avoid locking of the wheel and at the same time maximize the friction coefficient between the tire and runway surface, thus minimizing the stopping distance. It may also take passenger comfort as a secondary objective into consideration.

As explained in the Section 5.1.2, the anti-skid algorithm implemented in SIMPACK

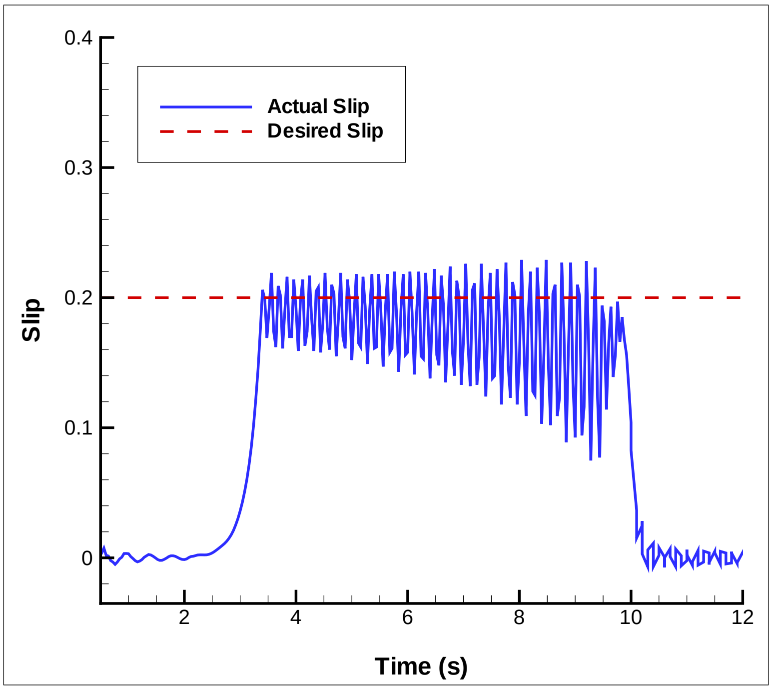


Figure 6.1: ABS algorithm using slip optimization

is used for the optimization of the slip value while braking. Figure 6.1 shows how the slip is optimized to get the maximum amount of braking possible without the wheel skidding or locking. Dynamic braking, explained in Section 5.1.1, has also been examined in the work as an alternative braking algorithm. This approach is normally used in flight tests as crude braking action. Although it is clear that this kind of braking will not be ideal from the passenger comfort and smooth braking point of view, it was implemented to stress the point that braking algorithm based on slip-optimization principle is a better alternative. Further, this algorithm based on slip-optimization is improved by adding hydraulic dynamics, as explained in Section 5.1.2 (called as Modified Anti-Skid Braking Algorithm), and is used to study the brake and landing gear interaction. The following results clearly show that the anti-skid braking algorithm is a better alternative for braking as it provides better passenger comfort and reduced friction-induced vibration, and is stable compared to the dynamic braking. Along with the shorter braking distance, as a secondary objective the anti-skid system may also be designed to improve the passenger comfort. The following results, See Figure 6.2 and 6.3, show how anti-skid algorithm reduces forces in X direction at the main landing gear attachment and is stable when it comes to acceleration at the attachment point. It shows that with the ABS algorithm based on slip-optimization principle smooth braking is achieved with the reduced strut vibrations and in turn the better passenger comfort.

6.1.2 Friction-induced Vibrations

Gear walk, as explained in the Section 2.2, is cyclic fore-aft motion of the landing gear assembly about a normally static vertical strut-center line. Gear walk instability is illustrated by the time histories of gear-deflection, brake torque, and wheel-tire footprint (speed) as shown in Figure 6.4 and 6.5.

For this multi-body simulation a two mass model of a flexible landing gear at the attachment point is used. Braking action is initiated at the end of 2 second rolling in forward direction until the desired speed is achieved. Though initially the gear deflection increases when brakes are applied the amplitude is does not grow as compared to the dynamic braking due to the slip-optimization principle behind the anti-skid algorithm. Once the desired speed is achieved the deflection reduces very fast and is almost zero. This is a desired behavior compared to the chaotic vibrations produced during the dynamic braking simulations. The time history of gear deflection, shown in Figure 6.4, and wheel-tire footprint (speed) as shown in Figure 6.5 prove that with the help of anti-skid algorithm implemented in SIMPACK it was possible to detect the gear-walk phenomena and study the landing gear vibrations.

6.1.3 Modified Anti-Skid Braking Algorithm

Although the strut vibrations and comfort was improved with the anti-skid algorithm, the gear vibrations in longitudinal (termed as Gear Walk) and lateral directions (shimmy vibrations) were still high. This braking algorithm did not also have the hydraulic dynamics included, the next logical step was to implement an improved

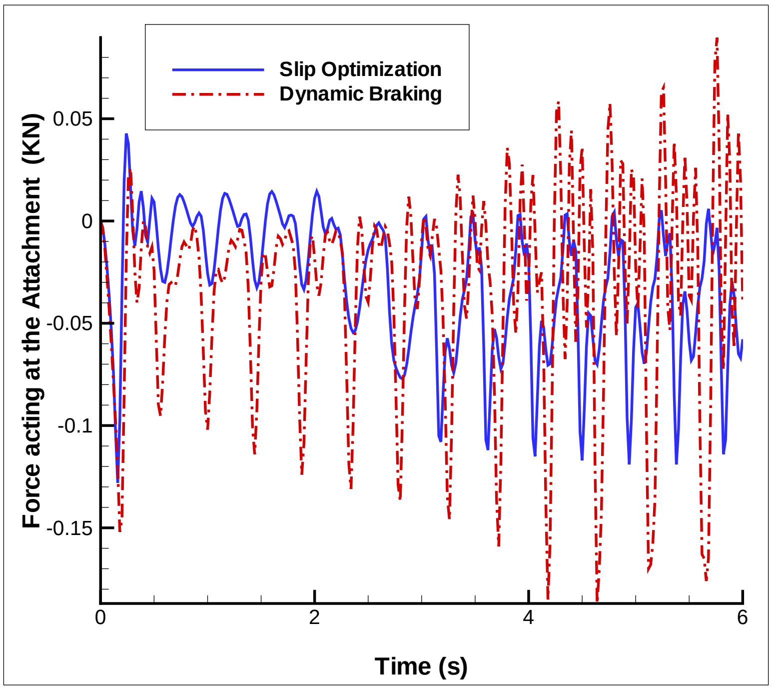


Figure 6.2: Slip optimization v/s dynamic braking

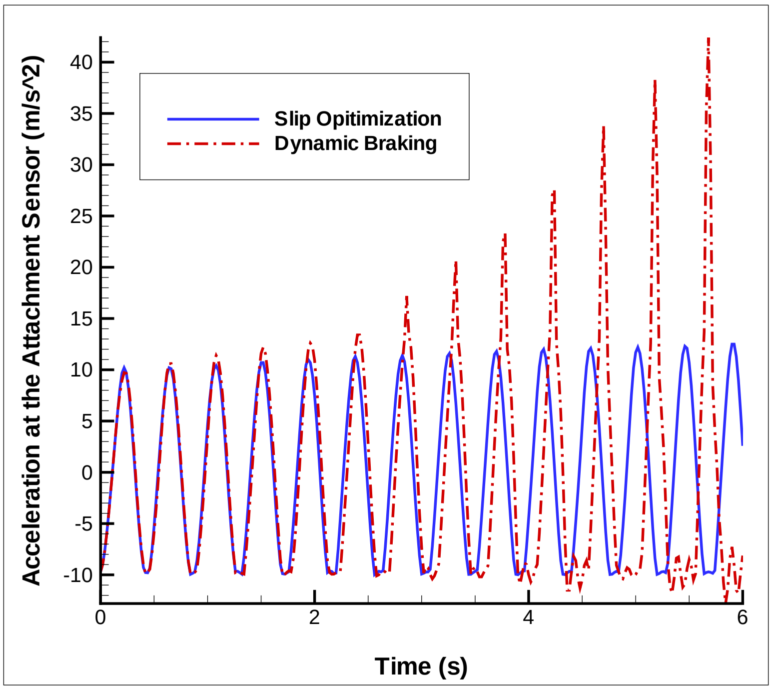


Figure 6.3: Slip optimization v/s dynamic braking

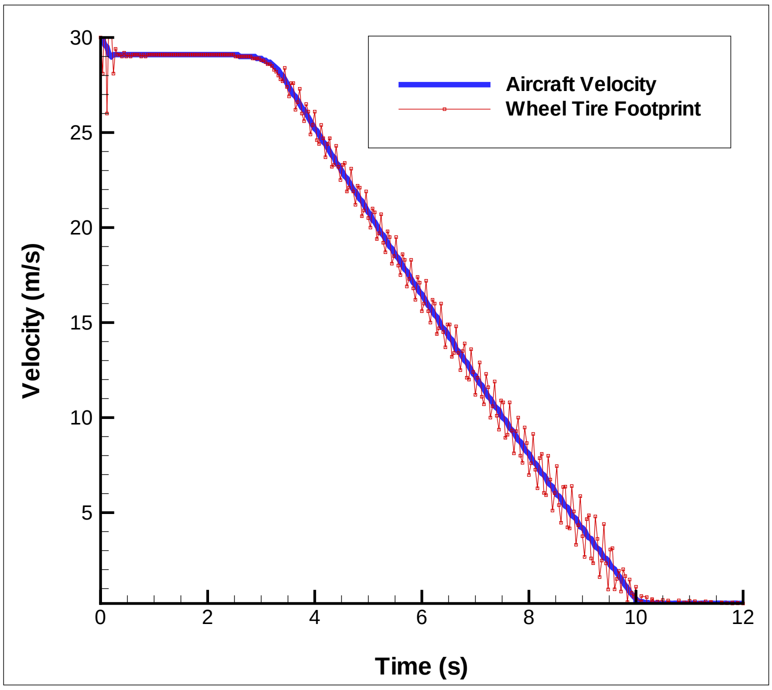


Figure 6.4: Slip optimization v/s dynamic braking

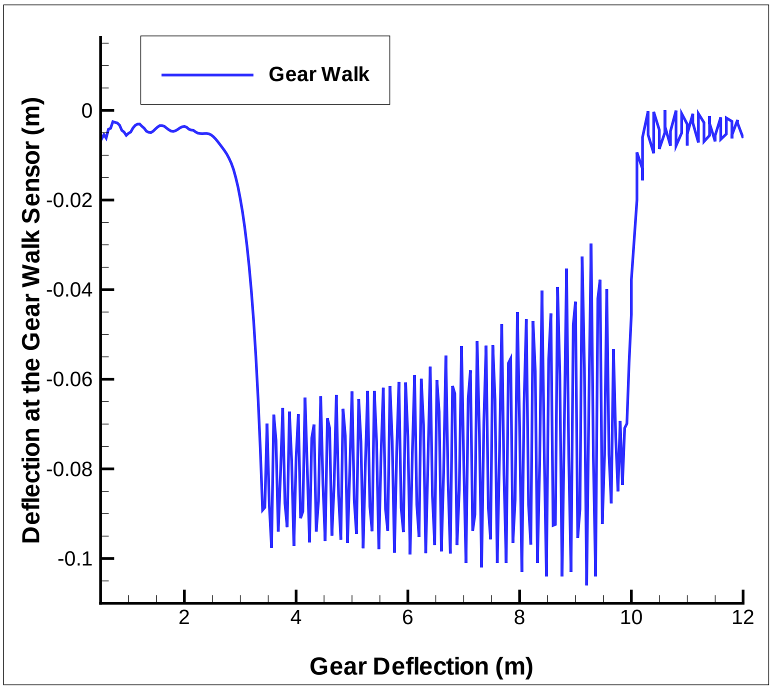


Figure 6.5: Gear walk instability with ABS using slip optimization

braking algorithm using hydraulics. The simulations showed vast improvements in the stability and the gear vibrations. Figure 6.6 shows the improved stability and reduced deflection at the gear walk sensor when sophisticated brake system with hydraulic lag is used.

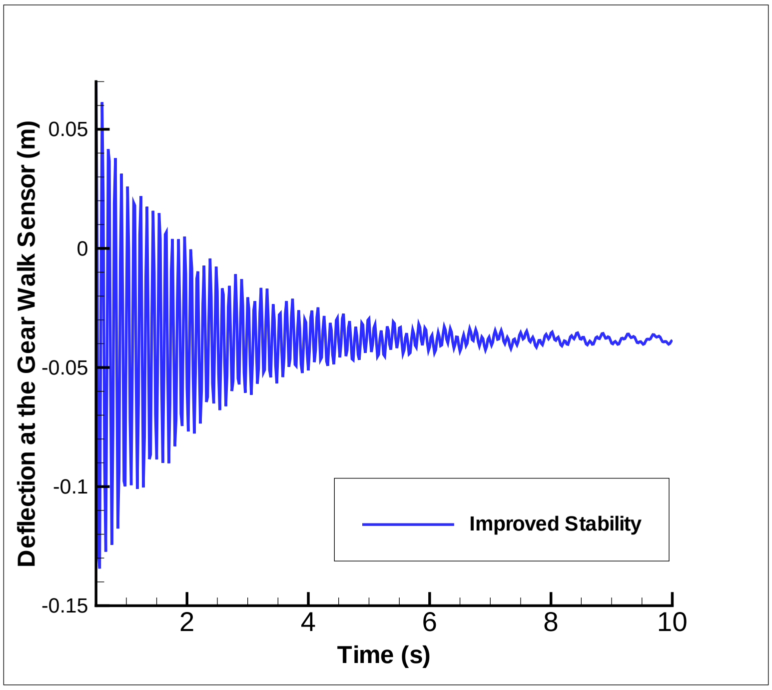


Figure 6.6: Improved stability with modified braking algorithm

6.1.4 Effect of Geometrical Parameters of Hydraulic Line

The dynamics of the hydraulic system plays an important role in the brake dynamics and its interaction with the landing gear vibration. A two mass model of a flexible main landing gear was used for the simulation of this interaction. Equation 5.3 shows the relationship between the pressure drop and the critical parameters of the hydraulic line, namely thickness and length of the pipe and the viscosity. Simulations showed that the thickness is more critical geometric parameter as it also affects the Reynolds Number, R_e and velocity of the fluid inside the pipe, v. The results obtained in Figure 6.8 show that increasing the thickness of the pipe may produce undesired vibrations at the gear walk sensor and at the attachment point. Increasing the length of the pipe does affect the the pressure drop but not the pressure input at the brake disk which is quite high compared to the pressure drop. In turn the braking force is not much affected.

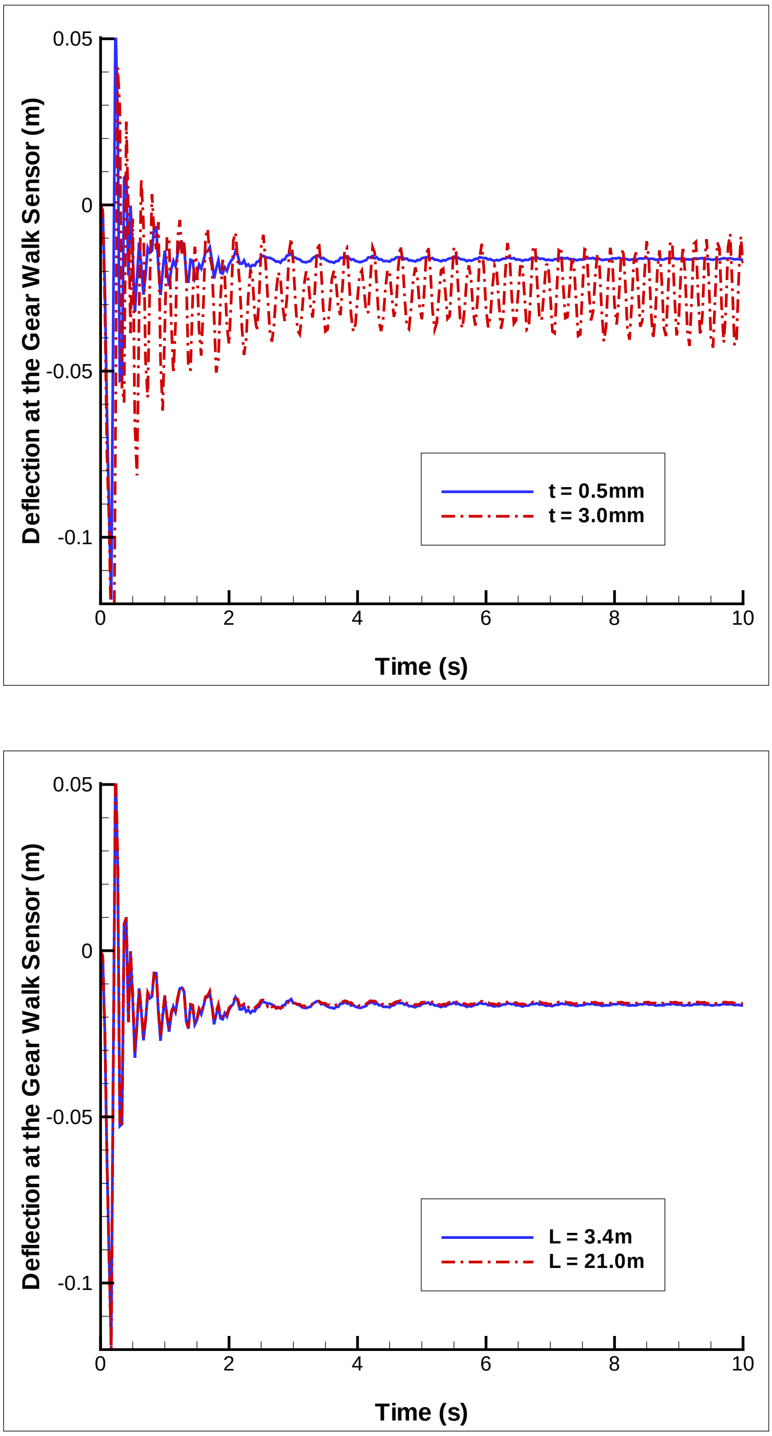


Figure 6.7: Effect of geometrical parameters on stability, Top: Thickness of hydraulic line, Bottom: Length of hydraulic line

6.1.5 Shimmy

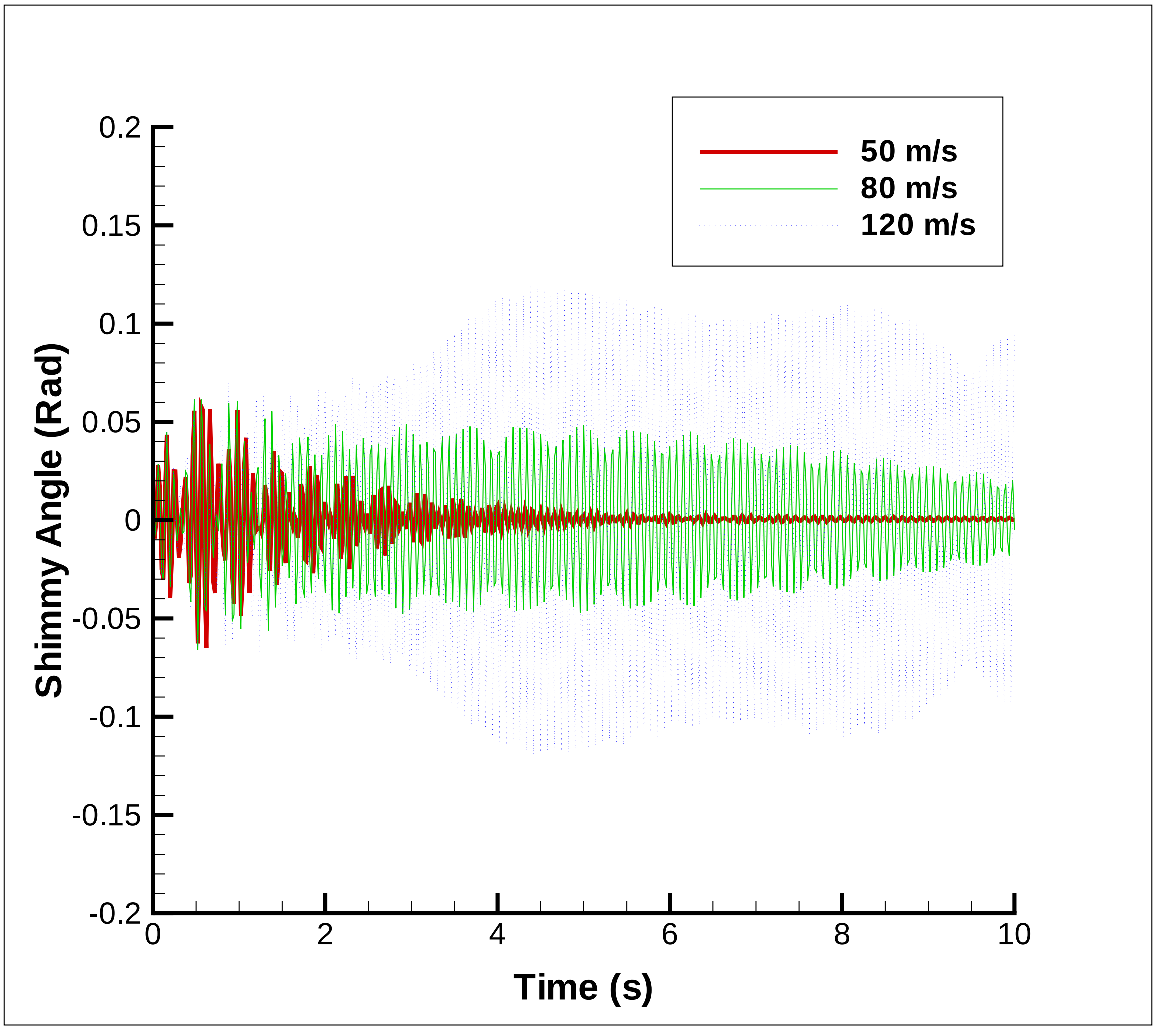


Figure 6.8: Shimmy at different rolling speeds

The oscillations exhibited by the steerable wheels is popularly known as "shimmy". Shimmy is also the term used to describe the self-induced swiveling motion of the gear of an airplane. Pneumatic tires are used for obtaining better road-holding and comfort. The vertical and lateral elasticity of the tire introduce additional degrees of freedom of motion, which are coupled with the angular motions of the wheel about the swivel axis [50]. Such a coupling may lead to the occurrence of an oscillatory instability of the stationary rectilinear motion. In case of aircraft landing gear this may be caused by the landing gear structure and interaction of the brake and gear. These vibrations are usually in range of 10 to 30 Hz, and may assume grave proportions and cause failure of mechanical components or result in loss of control of the vehicle.

The most important task of these MBS simulations was to be able to detect the shimmy oscillations. Flexible landing gear of a two-mass model was prepared as explained in Section 4.2.3. Simulations were done with the non-linear model for forward velocities between 20 to 120 m/s. Lateral deflections at the shimmy sensor (in Y-axis) and angular deflection around Z-axis are regarded as representative quantities of interest and have been plotted for the time-simulations, See Figure 6.8. It clearly shows that the shimmy oscillations were detected using this setup and simulation cases.

6.1.6 Effect of Structural Parameters

It was one of objectives of these simulations to study the effect of various parameters (geometric dimensions, tire mass, trail length etc.) on the gear vibrations. Important parameters were changed by 25% or by certain amount to see if it has any effect on the stability. The purpose of this section is to show how much influence each of the system parameters has on shimmy. Time simulations and eigen value analysis were done at the forward velocities between 20 and 120 m/s in steps of 10 m/s.

It is known that for fixed parameters, the gear may become unstable and subsequently develop shimmy as the aircraft taxiing velocity is increased beyond a critical point [43]. After the eigenvalue analysis of the system real part of the eigenvalue was plotted against the velocity. Figure 6.10 shows a plot of maximum real part of the eigenvalue against the taxiing velocity. Time simulations were performed with velocities between 20 to 120 m/s keeping all other parameters constant. It clearly seen from the Figure 6.10 that as the velocity is increased the real part of the eigenvalue starts reducing and becomes zero at $V = 115.2$ m/s, above which system becomes unstable. Figure 6.9 also shows that the natural damping of the system is reducing and reaches zero as the velocity is increased. After time simulations were done for the given configuration, shimmy angle was plotted for the 50, 80, and $120m/s$ rolling speeds. Figure 6.8 shows the shimmy oscillations for this simulation. We clearly see chaotic vibrations at the higher speeds as expected.

As a second parameter, wheel size and mass were taken into consideration. To study the effect of wheel size (Tire mass and Moment of Inertia) on stability three different Nastran models were prepared with $\pm 25\%$ difference to that of default value. Stability boundaries were plotted. Eigen analysis was done at different velocities and natural damping values were plotted against the roll velocities from $20m/s$ to $120m/s$. Figure 6.11 show the effect of wheel size and tire mass. Wheel mass and moment of inertia was increased by 25% and it was found that a larger and heavier wheel is undesirable. When wheel mass was increased by 25% the resulting system has lower natural damping and is less stable.

The ratio between lateral and yaw motion at the wheel axle center upon application of a moment about the vertical axis, is defined as effective trail. In the case of landing gears with lateral support by means of a side stay attachment a significant contribution is added. The total lateral deflection at the wheel axle center now consists of two components, namely, the torsional deformation about the strut center line times the mechanical trail plus contribution of the strut lateral displacement. In the case of landing gear under consideration this will result in the effective trail being larger than the mechanical trail. One of the factors playing an important role in stability is the mechanical trail length, e. So the placement of side stay plays an important role here. To study the effect of mechanical trail on stability three different Nastran models were prepared with $\pm 25\%$ difference in mechanical trail than the default value. Stability boundaries were plotted for three different trail lengths. Eigen analysis was done at different velocities and natural damping values were plotted against the roll velocities from $20m/s$ to $120m/s$. If the system can provide more damping than the critical value, then it can be said to be stable. From Figure 6.12, it is clear that for smaller trail length the system is more stable whereas when trail length is increased the system becomes less stable. Li [43] in his work has shown that the smaller trail length is desired from the stability point of view, the result obtained here shows a good agreement with it.

As next set of parameters wheel span and cant angle were take into consideration. Nastran models were prepared with $\pm 25\%$ difference in wheel span and with forward cant angle $+5\%$ than the default value. Stability boundaries were plotted for three different trail lengths. Eigen analysis was done at different velocities and natural damping values were plotted against the roll velocities from $20m/s$ to $120m/s$. Figure 6.14 show the effect of increasing the wheel span and cant angle of the gear in forward direction. It is seen that when the wheel span is increased by 25% of the default value, the natural damping of the system is reduced. Thus the resulting system is less stable. Whereas increase in cant angle of the gear improves the stability of the system which is in agreement with [12, 16].

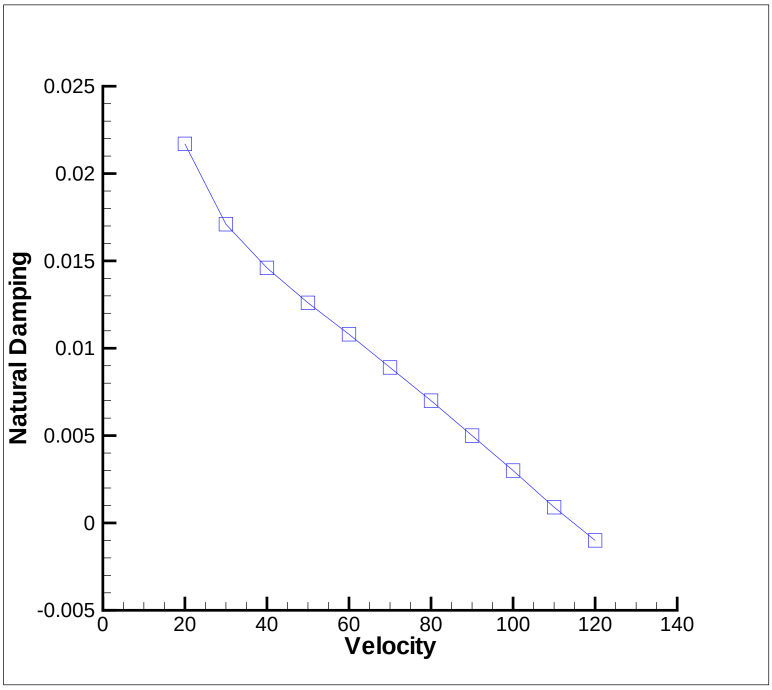


Figure 6.9: Natural damping of the angular motion of the MLG system

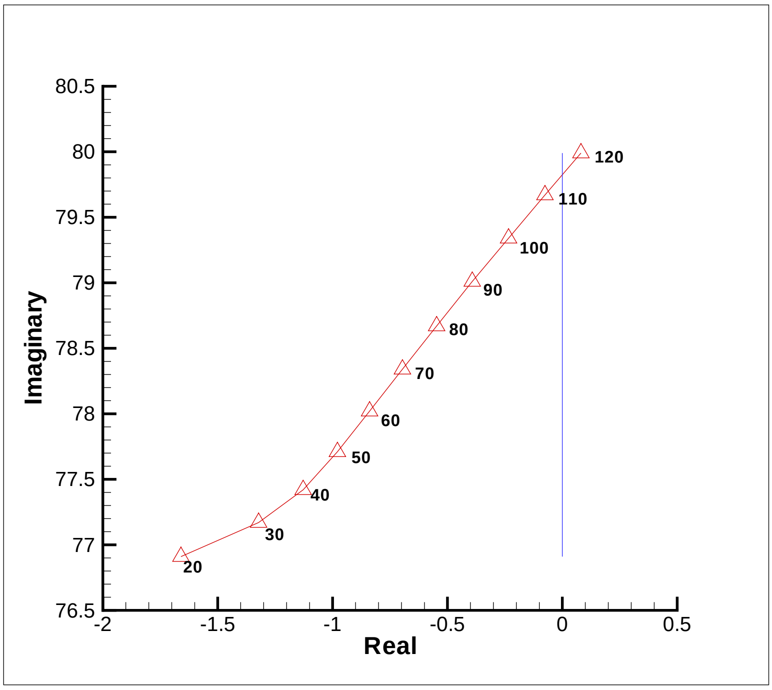


Figure 6.10: Argand diagram for different rolling speeds

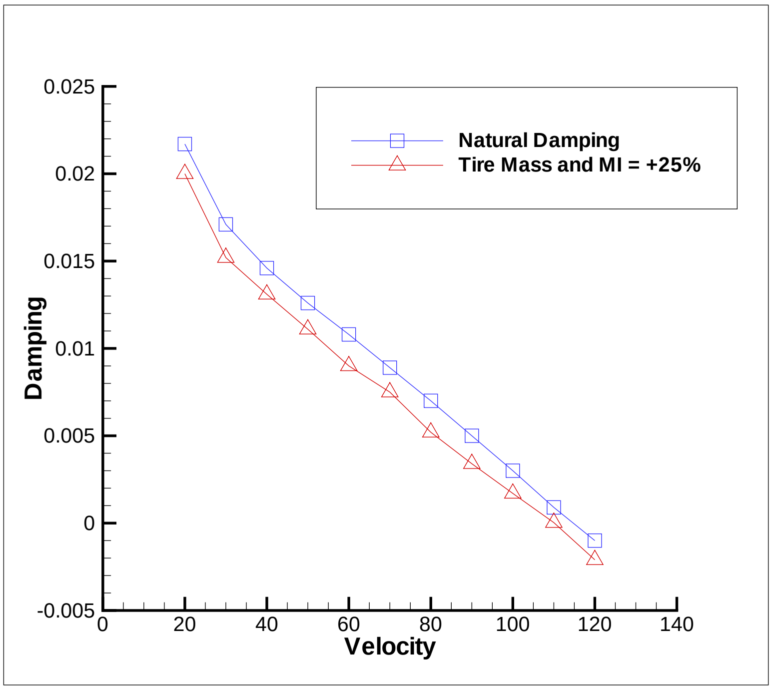


Figure 6.11: Effect of wheel size (Tire mass and MI)

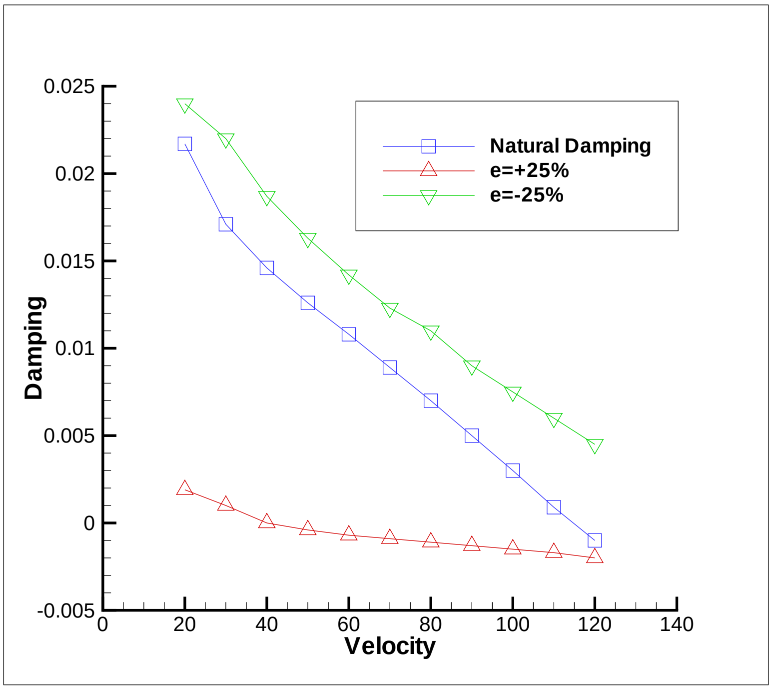


Figure 6.12: Effect of mechanical trail

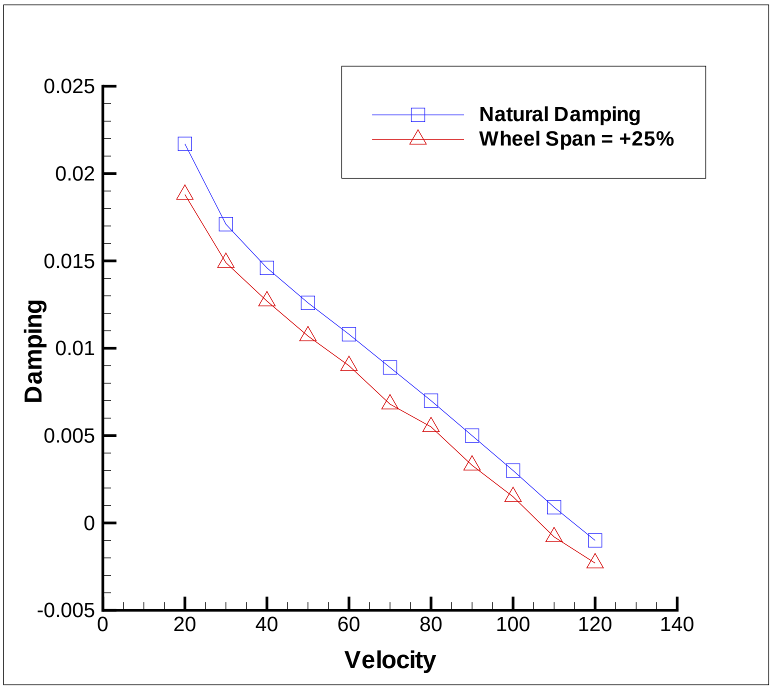


Figure 6.13: Effect of wheel span

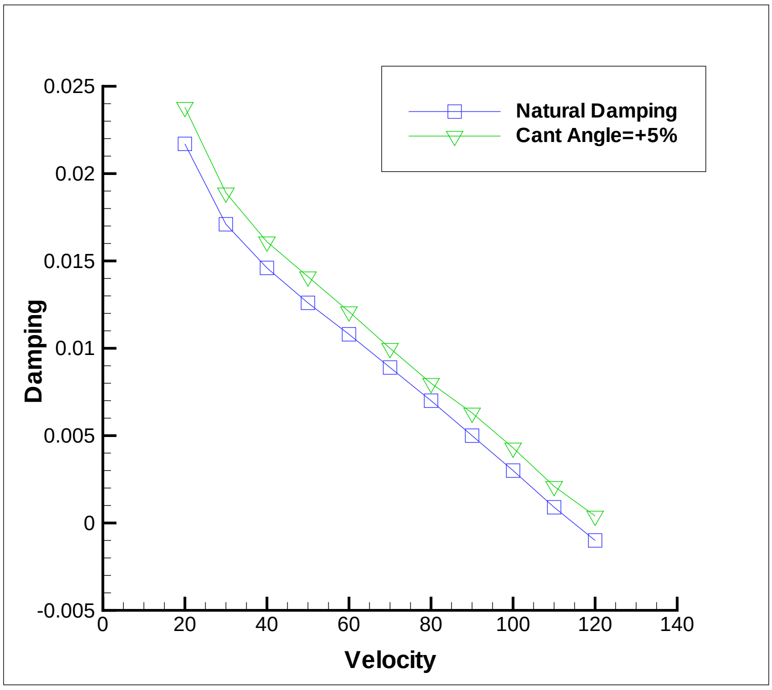


Figure 6.14: Effect of forward cant angle

6.1.7 Effect of Braking and Excitation on Shimmy

For studying the effect of excitation and braking on landing gear vibrations both the models are taken into consideration, explained in Section 4.2.3. The model one, as shown in Figure 4.8 is a single beam model. Shock strut travel inside the main-fitting is assumed to be constant during rolling. Whereas second model is prepared with variable travel of shock-strut inside the main-fitting. This model is further improved by means of free-play between the main-fitting and shock-strut. For the non-linear simulations the landing gear is excited by a load and a torque peak on the axle, please refer to Section 4.2.4. Free-play has been modeled as two additional rotational degrees of freedom in the joints between the piston and cylinder.

As proved earlier in Section 6.1.3, the new modified braking algorithm resulted in reduced strut vibrations in longitudinal direction. After the simulations were done to detect the lateral and yaw motions, which is defined as shimmy, it was found that the new algorithm also helps to lower the shimmy vibrations. Model with constant shock-strut travel is used for these simulations and brakes are applied after $1s$ till the forward velocity is reduced to zero or $10s$ simulation time. Simulations were done for different roll velocities from $20m/s$ to $120m/s$. The results plotted in Figure 6.15 show that the braking algorithm does not act as a source of additional vibrations and actually helps in reducing the shimmy vibrations. To validate this for the real life case, simulations were done for a model with variable shock strut travel and other non-linearities such as free-play and damper. Simulations were done at different rolling velocities from $20m/s$ to $120m/s$ for the model with free-play and without free-play. For comparison, simulations results are also plotted for the case when braking was applied and without braking. As mentioned earlier, braking helps to keep the vibrations low in both the cases. It has been mentioned in the earlier work by I.J.M. Besselink [3] that any changes in stiffness will affect shimmy stability. For example the equivalent yaw stiffness will be reduced by introducing yaw free-play; for a gear with positive mechanical trail this may result in shimmy vibrations. The free-play in lateral direction may actually reduce the the strut vibrations in some configurations. Various simulations were done using free-play values and it was seen that it does lead to additional vibrations, See the results plotted in Figure 6.16 and 6.17. However if you chose a model with negative trail and chose the stiffness values such that there is slightly more stiffness in lateral direction it clearly reduces the vibrations as seen in Figure 6.19 and 6.19. Further, Figure 6.20 braking algorithm does not have any undesirable effect on stability.

Another important parameter of the braking logic which affect the performance, both from the stability and braking point of view, is the hydraulic lag. As explained earlier in Section 5.1.4 and 5.1.3, this is defined by using basic equations of hydraulic losses and discrete-analogue filter which is available as standard routine in SIMPACK. In practice it is controlled by means of denominator coefficient. Figure 6.21 shows that the smaller values of coefficient lead to faster control of braking, however lead to larger vibrations at gear walk sensor. It may be possible to design the controller in such a way that these vibrations are reduced along with the improved braking performance.

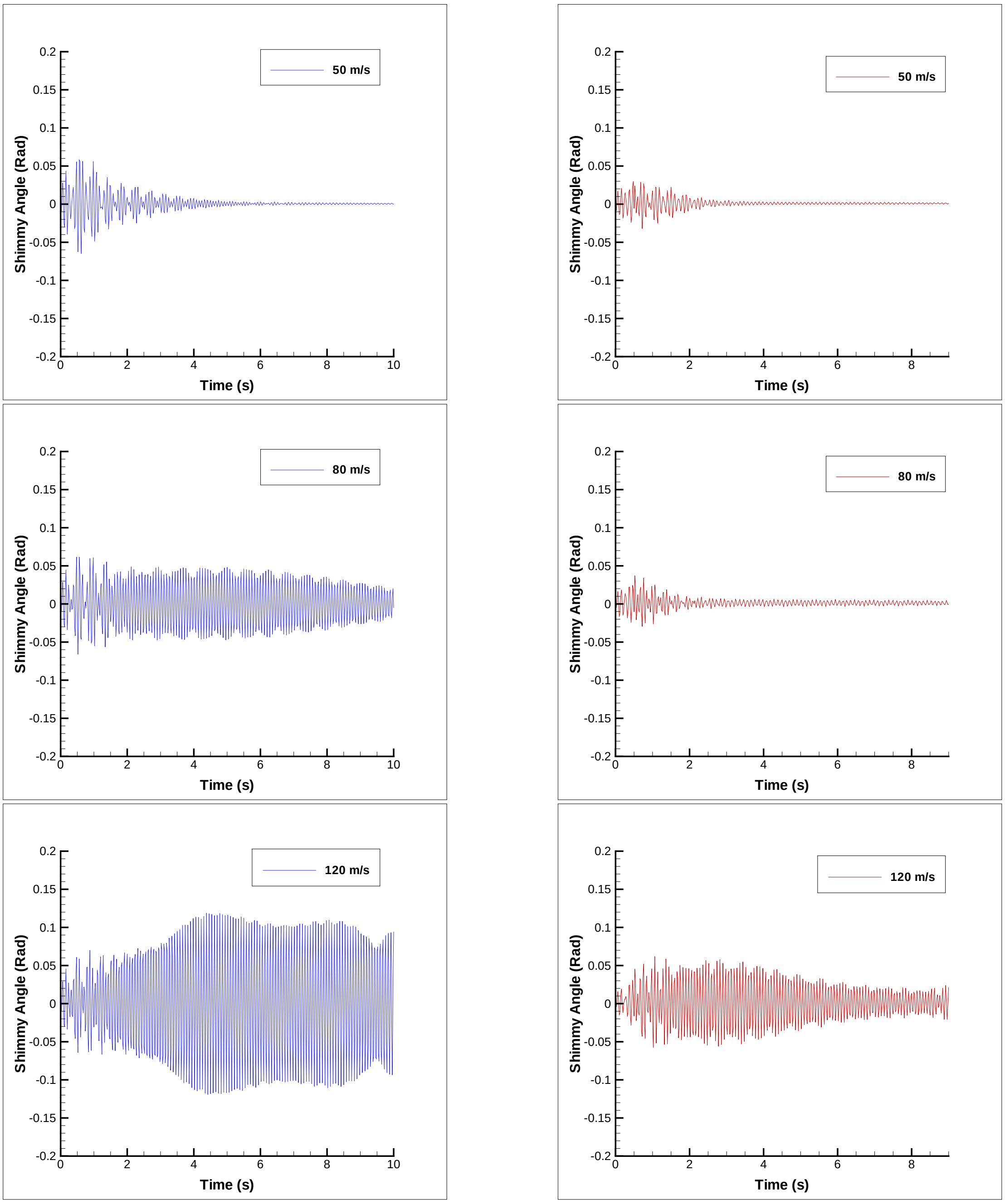


Figure 6.15: Landing gear shimmy at different rolling speeds , Left: No braking, Right:Braking with modified algorithm

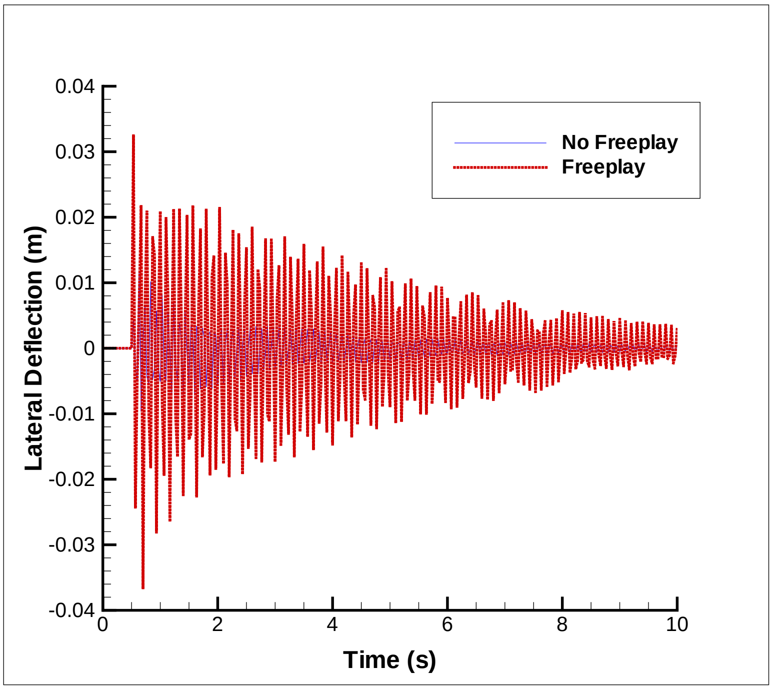


Figure 6.16: Shimmy vibrations; Effect of free-play

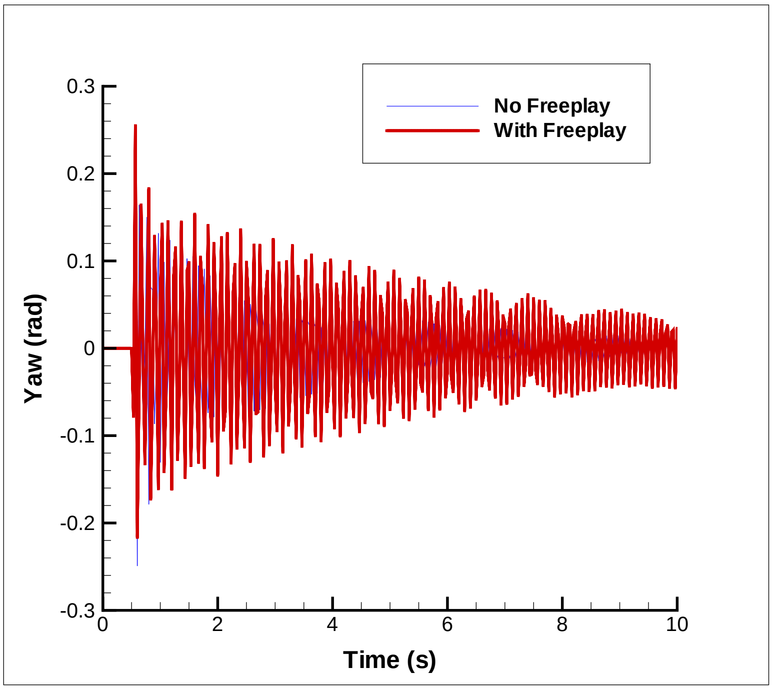


Figure 6.17: Shimmy Vibrations; Effect of free-play

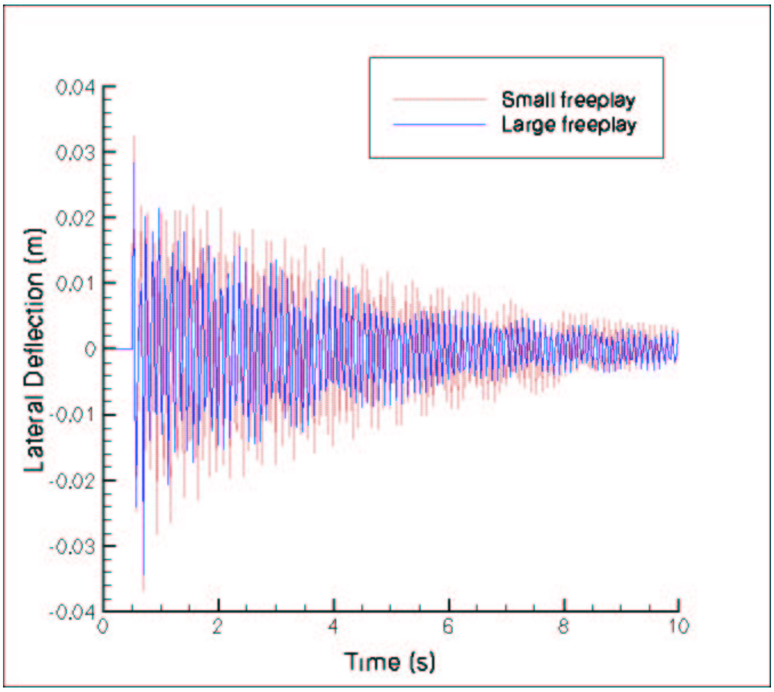


Figure 6.18: Shimmy vibrations; free-play comparison

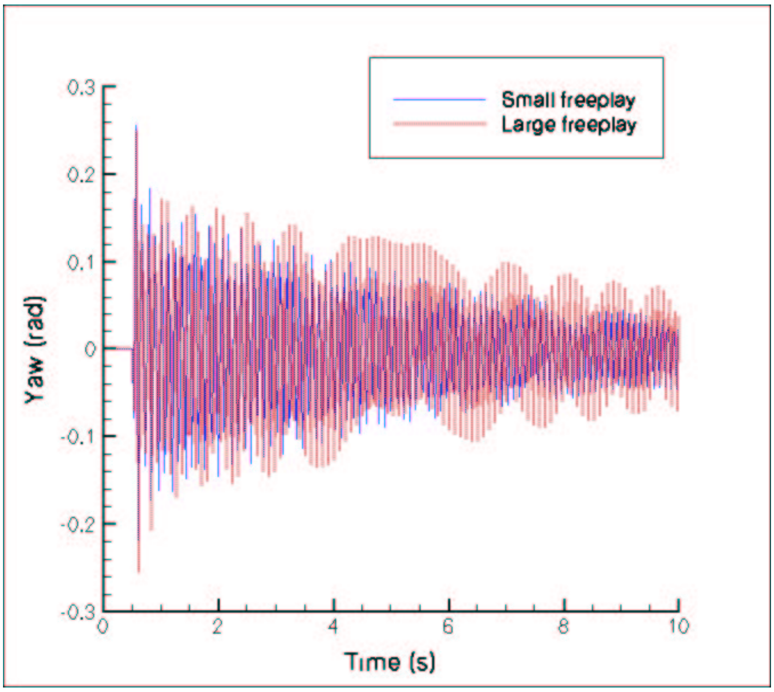


Figure 6.19: Shimmy vibrations; free-play comparison

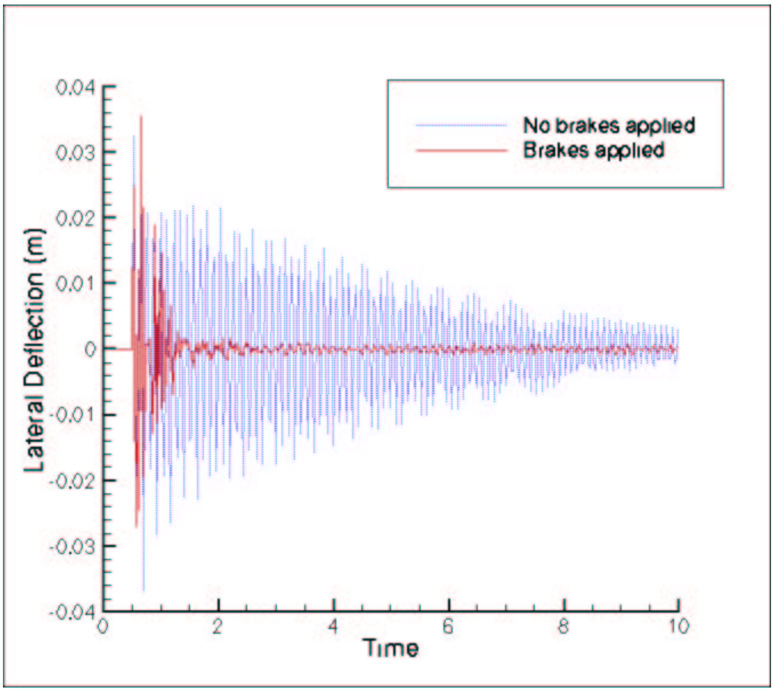


Figure 6.20: Shimmy vibrations; Effect braking with free-play

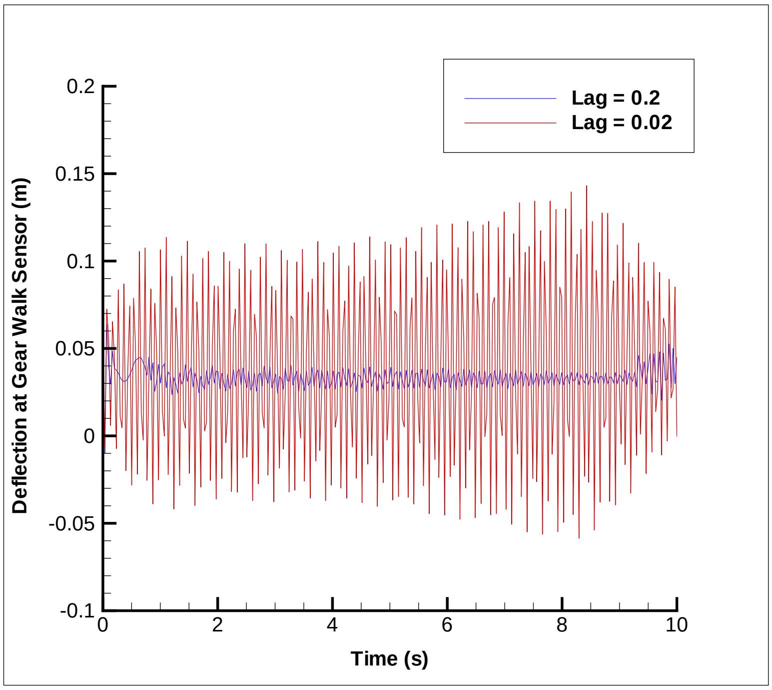


Figure 6.21: Deflections at gear walk sensor for different time lag

Chapter 7

Conclusions and Outlook

7.1 Conclusions

Aircraft spend a good part of its life on the ground. One of the main function of landing gear system is provision of means for the aircraft to maneuver on the ground, taxi and take off [12]. Landing gear dynamics, especially shimmy and brake-induced vibrations, is one of the problems faced today by the aircraft community. There have been many reports of fatal accidents due to vibrations generated by landing gear and brake interaction. Not only unsuitable combination of structural stiffness, damping, and pneumatic tire characteristics but also an unlucky combination of brake system design with the tire physics can produce a serious vibration problem [44]. It is important for this reason to put much effort in developing simulation models to predict parameter variations which may cause vibrations such as gear-walk and shimmy. Shimmy may be caused by a number of conditions such as low torsional stiffness, excessive free play in the gear, wheel imbalance, or worn parts. Brake-induced vibration includes conditions known as gear walk, squeal and chatter which are caused by the characteristic friction between the brake rotating and non-rotating parts.

In many cases, vibration problems may not be uncovered until physical prototypes are built and tested, adding considerable time and expense to the product development cycle. The use of commercially available computer-aided engineering tools made in the design phase by simulating the landing gear impact and rolling may be an alternative to detect the problem areas. In this work an adequate landing gear model was prepared using state-of-the-art flexible multi-body tools. Additional modules, such as tire and brake dynamics, were added to detect the important phenomenon such as gear walk and shimmy. Effect of different braking algorithm was also studied and a modified anti-skid method was proposed. Parameter study was conducted in order to study the effect of important structural and other parameters on landing gear vibrations.

As a first building block landing gear for the Liebherr regional aircraft was prepared as a multi-body model in SIMPACK and was further enhanced by the flexible model developed in NASTRAN. This flexible model was fairly accurate and showed eigen shapes and frequencies similar to that of the flexible model used at the Liebherr Aerospace for stress-analysis. The flexible model was imported in SIMPACK using the interface called FEMBS. The eigenfrequencies of interest were chosen for the dynamic analysis.

One of the important part of the work was to able to detect the vibrations, such as gear-walk and shimmy, in landing gear. Two different flexible models were used for this, one with constant shock-strut travel inside main-fitting and other with variable travel. A simple rolling simulation with excitation showed that for both the models it was possible to detect these potentially unstable phenomenon. Further different braking algorithms were compared. A simple anti-skid braking algorithm based on slip-optimization principle showed that the strut-vibrations were reduced and it provided better passenger comfort compared to that of dynamic braking. However, it still needed to be refined. A new modified version of anti-skid was designed which included hydraulic lag and dynamics. The new braking algorithm showed vast improvement in terms of containing the vibrations and providing comfort. With this new algorithm parameter study was conducted for the geometrical parameters of hydraulic line. It was found that the thickness of the pipe is more critical parameter than the length. It affects the brake pressure at the disk.

Shimmy is one of the important vibration mode of a gear in aircraft system which may affect the stability and comfort. As a next set of simulation different nonlinear time-simulations and eigen analysis (linearization) were done with the two different landing gear models mentioned above. Simulations were done with the nonlinear model for forward velocities between 20 to 120 m/s. Lateral deflections at the shimmy sensor (in Y-axis) and angular deflection around Z-axis have been plotted for the time-simulations and parameter study was conducted by means of eigenvalue analysis.

It is known that for fixed parameters, the gear may become unstable and subsequently develop shimmy as the aircraft taxiing velocity is increased beyond a critical point. It was demonstrated that the shimmy oscillations become increasingly unstable with the increase in rolling speed of an aircraft. As next set of parameter wheel size and mass were considered. After the eigenvalue analysis of the system real part of the eigenvalue was plotted against the velocity for two sets of wheel size and mass. Wheel mass and moment of inertia was increased by 25% than the default and it was found that a larger and heavier wheel is undesirable. The system had lower natural damping. One of the factors playing an important role in stability is the mechanical trail length. It was found our after similar investigation that the smaller trail length is desired from the stability point of view. It was seen that when the wheel span is increased by 25% of the default value, the natural damping of the system is reduced. Thus the resulting system is less stable. Whereas increase in cant angle of the gear improves the stability of the system. The parameter study done here shows good agreement with the earlier work done using different tools [12, 16].

As a next stage of the work effect of various non-linearities such as braking, excitation and free-play is taken into the consideration. It was found that the braking did not produce any undesirable oscillations and helped to damp out the vibrations produced due to the external excitation. The second model with variable travel of the shock-strut and free-play was used to study the effect of free-play on the stability. It was found that the effective free-play used in the lateral direction helps to reduce the shimmy vibrations.

Lag of the system is defined by using basic equations of hydralic losses and discrete-anlogue filter which is available as standard routine in SIMPACK. It was found that the smaller values of coefficient lead to faster control of braking, however lead to larger vibrations at gear walk sensor.

7.2 Outlook

With the help of the flexible model of a landing gear and different modules such as braking algorithm it was possible to detect the instabilities such as shimmy and gear-walk. However, this is just the beginning of what can be a very effective way of studying these phenomenon. Various items require special attention and detailed work.

- Landing Gear Configuration: In this work two different models were considered based on the shock strut travel. The landing gear itself was a simple T-type landing gear. Complex models may be prepared with configuration other than T-type such as one used in large civil and military transport aircraft. With the additional components such as bogie and properties it will be challenging and interesting to built accurate models and predict vibrations.

- Different Tire and Runway Parameters: Although effect of lateral forces and moments were taken into consideration in this work, it will be interesting to study the effect of other tire parameters such as inflation pressure, unbalance and irregularities. It is possible to use different surface friction values with the current setup. It is necessary to address the effect of flexibilities of runway.

- Brakes: Although effect of braking is taken into consideration it is important to find out if a particular combination of parameter may hit resonance with the natural frequencies of the structure and produce instability. The way to overcome it if occurred needs to be addressed too. Various controller parameters are also important. It was found that the lag of the system affect the braking performance and vibrations. It may be possible to design the controller in such a way that these vibrations are reduced along with the improved braking performance.

- Validation: It is very important to validate the results with the experimental or field-test data.

It is important to put up guidelines to design the vibration-free landing gear. It is also important to design other parts of the landing gear system such as braking system so that it does not produce vibrations in the system or excite it. Though this work contribute in a way to understand these unstable phenomenon and present a newer tool, Flexible Multi-body Dynamics, to investigate them and the parameters which affect it, more work needs to be done. It is important to study different types of gears and how braking and other non-linearities may affect their stability. This will help to understand more about these instabilities and design vibration free landing gear system with confidence.

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Appendix A

Discrete or Analog Filter by Transfer Function F(w) in SIMPACK

This element serves as submodule in SIMPACK "*control loops*", and is filtering up to 3 analog or discrete input signals with the same filter. The mode of time calculation, continuous or time-discrete, is given by the force parameter force.par(2). With the flag force.par(3)=1 (conversion between continuous and discrete) the user can also use the original values of the matrices in the other mode. The filter is defined by a rational transfer function $F(w)$, given by a numerator and denominator polynom up to order 6.

For the time-continuous case the transfer function F(s) is

$$F(s) \;=\; \frac{b_{c_0} + b_{c_1}\, s + \ldots + b_{c_{nb}}\, s^{n_b}}{a_{c_0} + a_{c_1}\, s + \ldots + 1\, s^{n_a}} \quad \text{with} \quad \begin{array}{l} 0 \le n_b \le n_a \\ 1 \le n_a \le 6 \end{array}$$

and for the time-discrete case

$$F(z) \;=\; \frac{b_{d_0} + b_{d_1}\, z + \ldots + b_{d_{nb}}\, z^{n_b}}{a_{d_0} + a_{d_1}\, z + \ldots + 1\, z^{n_a}} \quad \text{with} \quad \begin{array}{l} 0 \le n_b \le n_a \\ 1 \le n_a \le 6 \end{array}$$

The order of the denominator must be larger than 0, while the order of the numerator may also be 0, but not larger than the order of the denominator.
The coefficient of the highest exponent of the denominator is in both modes always 1 and is not required as input parameter.

With a flag, force.par(2), the user is able to work either in the time-continuous case with the integration of differential equations of order n_a or with n_a algebraic equations in the time-discrete case.

A.0.1 Input variables

The input variables are outputs of the sensors (measurements, state variables, etc) and of AD-converters.

A.0.2 Force element states

The number of state variables is dependent on the number of input variables and on the order of the transfer function.
Per input variable a maximum of 4 differential equations can be possible. That means a maximum of 12 differential equations per each call of type 140.

The following table gives an overview about the given values and the calculation mode. If discret values are given and the calculation should be analogous a conversion has to be done (=1 conversion from analogous to discrete and vice versa;=2 conversion in block structure analogous - analogous; =3 conversion in block structure discrete - analogous).

Values given	conversion = 0: off > 0: on	calculation analogous = 0 discrete = 1
analogous	0	0
discrete	1	0
discrete	0	1
analogous	1	1
analogous	2	0
discrete	3	0

A.0.3 Equations:

For $n_b = n_a = 4$ the system of differential equations to be solved has the general form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -a_{c_3} & 1 & 0 & 0 \\ -a_{c_2} & 0 & 1 & 0 \\ -a_{c_1} & 0 & 0 & 1 \\ -a_{c_0} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} b_{c_3} - a_{c_3} b_{c_4} \\ b_{c_2} - a_{c_2} b_{c_4} \\ b_{c_1} - a_{c_1} b_{c_4} \\ b_{c_0} - a_{c_0} b_{c_4} \end{bmatrix} u_f$$

$$y = x_1 + b_{c_4} u$$

The discrete algebraic equations have a very similar form:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_{i+1} = \begin{bmatrix} -a_{d_3} & 1 & 0 & 0 \\ -a_{d_2} & 0 & 1 & 0 \\ -a_{d_1} & 0 & 0 & 1 \\ -a_{d_0} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_i + \begin{bmatrix} b_{d_3} - a_{d_3}\, b_{d_4} \\ b_{d_2} - a_{d_2}\, b_{d_4} \\ b_{d_1} - a_{d_1}\, b_{d_4} \\ b_{d_0} - a_{d_0}\, b_{d_4} \end{bmatrix} u_{f_i}$$

$$y_i = x_{1_i} + b_{d_{n_a}}\, u_{f_i}$$

The third input parameter, force.par(3), allows the calculation with the transfer function which is given in the other mode. The conversion of the transfer function is performed with the approximation of Tustin [22]:

$$\text{continuous} \Longrightarrow \text{discrete}: \quad s = \frac{2}{t_s}\,\frac{z-1}{z+1}$$

$$\text{discrete} \Longrightarrow \text{continuous}: \quad z = \frac{s + \frac{2}{t_s}}{-s + \frac{2}{t_s}} \qquad t_s\text{... sampling time}$$

More information about this approximation method is given in literature about digital filters and digital control, for example Gene F. Franklin and J. David Powell, 'Digital Control of Dynamic Systems', Addison-Wesley, [22].

In the case of transfer functions of higher order the values of the coefficients may differ in such relations that coefficients like the '1' , given in the general form above, might be not considered in the calculations. To avoid such numerical eliminations in the time-continuous mode a second form is offered to the user by option force.par(3) = 2 (original transfer function is given in time-continuous mode), respectively by option force.par(3) = 3 (original transfer function is given in time-discrete mode):
The transfer function is separated into several subsequent transfer functions of order 2, of which the size of the coefficients should be in a range well-suited for numerical handling.
To obtain this structure the numerator and denominator polynomial is factored, represented by the eigenvalues of the polynomials. The transfer function F(s) has the form

$$F(s) = \frac{(s-z_1)\,(s-z_2)\cdots(s-z_{nb})}{(s-p_1)\,(s-p_2)\cdots(s-p_{na})}$$

Then the n_k 2 $\times$ 2 - blocks are formed by products of the terms with conjugate complex eigenvalues or, if conjugate complex eigenvalues do not exist, of terms with two real eigenvalues. If the order of the denomiator polynomial is odd, the last block has the order 1. As the numerator polynomial can be of less order than the denominator polynomial the order of the numerator polynomials of the last blocks can be of order 0.

$$F(s) = \frac{b_{1_0} + b_{1_1}\, s + b_{1_2}\, s^2}{a_{1_0} + a_{1_1}\, s + s^2} \cdot \frac{b_{2_0} + b_{2_1}\, s + b_{2_2}\, s^2}{a_{2_0} + a_{2_1}\, s + s^2} \cdots \frac{b_{nk_0} + b_{nk_1}\, s}{a_{nk_0} + s}$$

This structure means a sequence of blocks which have themselves (until the order of 2) the same form of differential equations as given in the general form above. The input value into the subsequent block is the output value of its predecessor. So the representation of the whole system of differential equations would show a systemmatrix A in upper Hessenberg form. In this structure the first two differential equations belong to the last block , the next equations to its predecessor and so on. If the order of the numerator polynomial is less than the denominator the input vector $\underline{b}_u$ shows only for the rows of the first block no-vanishing coefficients. In the other case the input value u_f is input of the differential equations of all blocks.

Large relations between the coefficients of numerator and denominator may cause numerical problems. With force.par(32) = 1 the coefficients of the numerator are automatically scaled so that the relation $0.1 \leq \mid \frac{b_o}{a_0} \mid \leq 10.$.

Appendix B

Connection Element and Function Generator in SIMPACK

Two work values of sensors, AD-converter or filter stage 1 amplified by a factor can be connected in a different way , as Fig. B.1 shows. The result z is the input in a function, which can be a limitation, an input function defined by the user, the absolute value of z, a signum - function etc.. Presently the user can choose among ten different functions. The sum of the function value with a constant zero shift, defined by force.par(13) is the work value ov(1).
The first input value is the work value of a control element defined by force.par(1) and force.par(2), the second input value is the work value of a control element defined by force.par(4) and force.par(5). If force.par(1) or force.par(2), respectively force.par(4) or force.par(5) is not defined, the time replaces the according input.

According to the value of force.par(7), the connection yields

- only the first amplified input value $z = c_1 u_1$, (force.par(7)=0)
- the addition $z = c_1 u_1 + c_2 u_2$, (force.par(7)=1)
- the product $z = c_1 u_1 \cdot c_2 u_2$, (force.par(7)=2) or
- the division $z = \frac{c_1 u_1}{c_2 u_2}$, (force.par(7)=3).

With force.par(9) the desired function is chosen. For this function a further parameter c_F = force.par(8) is used. Actually the user has 10 different possibilities:

1. force.par(9) = 0 : The function value is only the amplified input value z:

$$ov(1) = c_F z$$

2. force.par(9) = 1 : Limitation of z to the value c_F :

$$\mid ov(1) \mid > c_F + force.par(13) \quad \longrightarrow \quad ov(1) = c_F sign(z) + force.par(13)$$

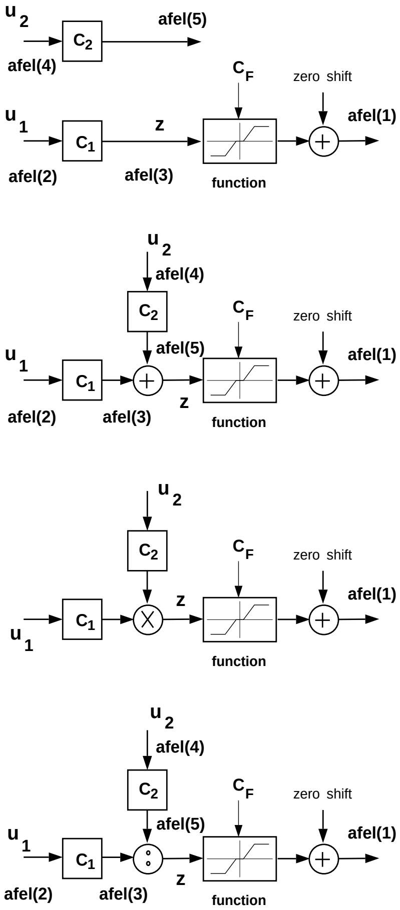

Figure B.1: Possible Structures of the Filter

3. force.par(9) = 2 : The function value is calculated by interpolation of an input function defined by the user.

$$ov(1) \;=\; fct(\, c_F\, z\,) \;+\; force.par(13)$$

force.par(10) is the name of the input function and force.par(11) defines the kind of interpolation, linear (=0) or spline interpolation (=1). Choosing linear interpolation the user has the possibility to smooth the transition from one segment to the next by defining a smoothing range with force.par(12).

4. force.par(9) = 3 : The function value is the absolute value of z amplified by c_F.

$$ov(1) \;=\; c_F \;\mid ov(1) \mid + force.par(13)$$

5. force.par(9) = 4 : The function value is the signum-function:

$$ov(1) \;=\; c_F\, sign(z) \;+\; force.par(13)$$

6. force.par(9) $=5$: The function value is the rms-value of the input function $c_F\, z$:
   With two integrations the mean value z_m and the variances σ^2 are calculated at each time T:

$$
\begin{array}{lllll}
x_1(T) & = & \int\limits_0^T c_F\, z dt & ov(6) \equiv z_m(T) & = \frac{x_1(T)}{T} \\
x_2(T) & = & \int\limits_0^T (c_F\, z)^2\, dt & ov(7) \equiv \sigma^2(T) & = \frac{x_2(T)}{T} - z_m^2(T)
\end{array}
$$
$$
\text{rms - value}: \; ov(1) \equiv rms(T) = \sqrt{\sigma^2(T)}
$$

7. force.par(9) $=6$: The input value z is restricted by a modulo-function with the parameter $c_F =$ force.par(8)

$$ov(1) = mod(z, c_F) + force.par(13)$$

8. force.par(9) $=7$: This function represents the time delay by Padé-Approximation, a transfer function of 2^{nd} order where the delay time T_D is defined by c_F (force.par(8)):

$$T_D(s) = \frac{\frac{12}{T_D^2} - \frac{6}{T_D}\, s + s^2}{\frac{12}{T_D^2} + \frac{6}{T_D}\, s + s^2}$$

   This transfer function yields two differential equations:

$$
\begin{array}{lll}
\dot{x_1} & = & -\frac{6}{T_D}\, x_1 + x_2 - \frac{12}{T_D}\, z \\
\dot{x_2} & = & -\frac{12}{T_D^2}\, x_1 \\
ov(1) & = & x_1 + z + force.par(13)
\end{array}
$$

9. force.par(9) $=8$: This function represents a comparator with a switch, fig. B.2.

$$
\begin{array}{llll}
\text{For} & c_1\, u_1 \geq c_2\, u_2 & \Longrightarrow & ov(1) = c_3\, u_3 + force.par(13) \\
\text{for} & c_1\, u_1 < c_2\, u_2 & \Longrightarrow & ov(1) = c_4\, u_4 + force.par(13) \quad .
\end{array}
$$

   So this function element differs in its structure from those given in the figures B.1 : The output is either $c_3\, u_3$ or $c_4\, u_4$. Only the switching logic depends on the values $c_1\, u_1$ and $c_2\, u_2$.

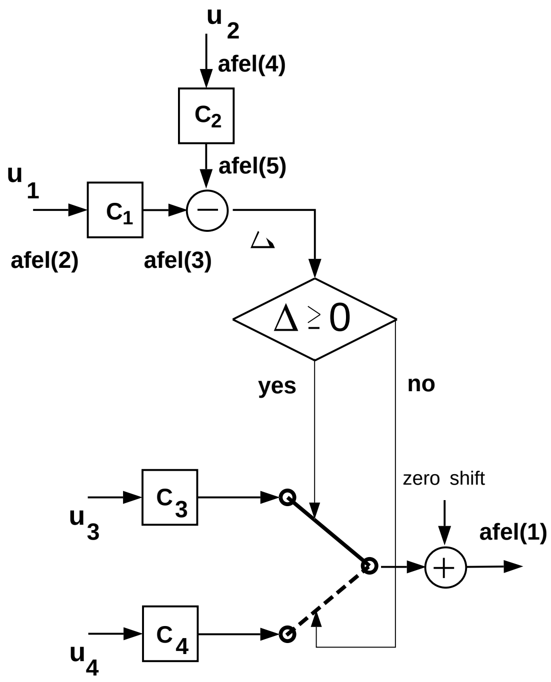

Figure B.2: Function Comparator and Switch

10. force.par(9) = 9 : This function is the evaluation of a two-dimensional parameter array where the function value is dependent on two input parameters x_1 and x_2, fig. B.3. The two-dimensional parameter array must be stored in the directory
$SIMPACK_MODEL/dat/par_array with an arbitrary name. This file name must be listed together with its integer code number and short description in the file
$SIMPACK_MODEL/dat/par_array/PAR_ARRAY_LIST.dat. An example is given at the end of this chapter.

    Input values to the parameter array are the values $c_1\, u_1$ and $c_2\, u_2$. The parameter array can be evaluated in three different ways:

    - 1. The normal evaluation y=fct(x_1,x_2) (force.par(22)=0): The input parameters $x_1 = c_1\, u_1$ and $x_2 = c_2\, u_2$ are given. The function y is evaluated. Output $ov(1) = y + force.par(13)$.
    - 2. The inverse evaluation x_1=fct(y,x_2) (force.par(22)=1): The function value $y = c_1\, u_1$ and the second input parameter $x_2 = c_2\, u_2$ are given. The fist input parameter x_1 is evaluated. Output $ov(1) = x_1 + force.par(13)$.

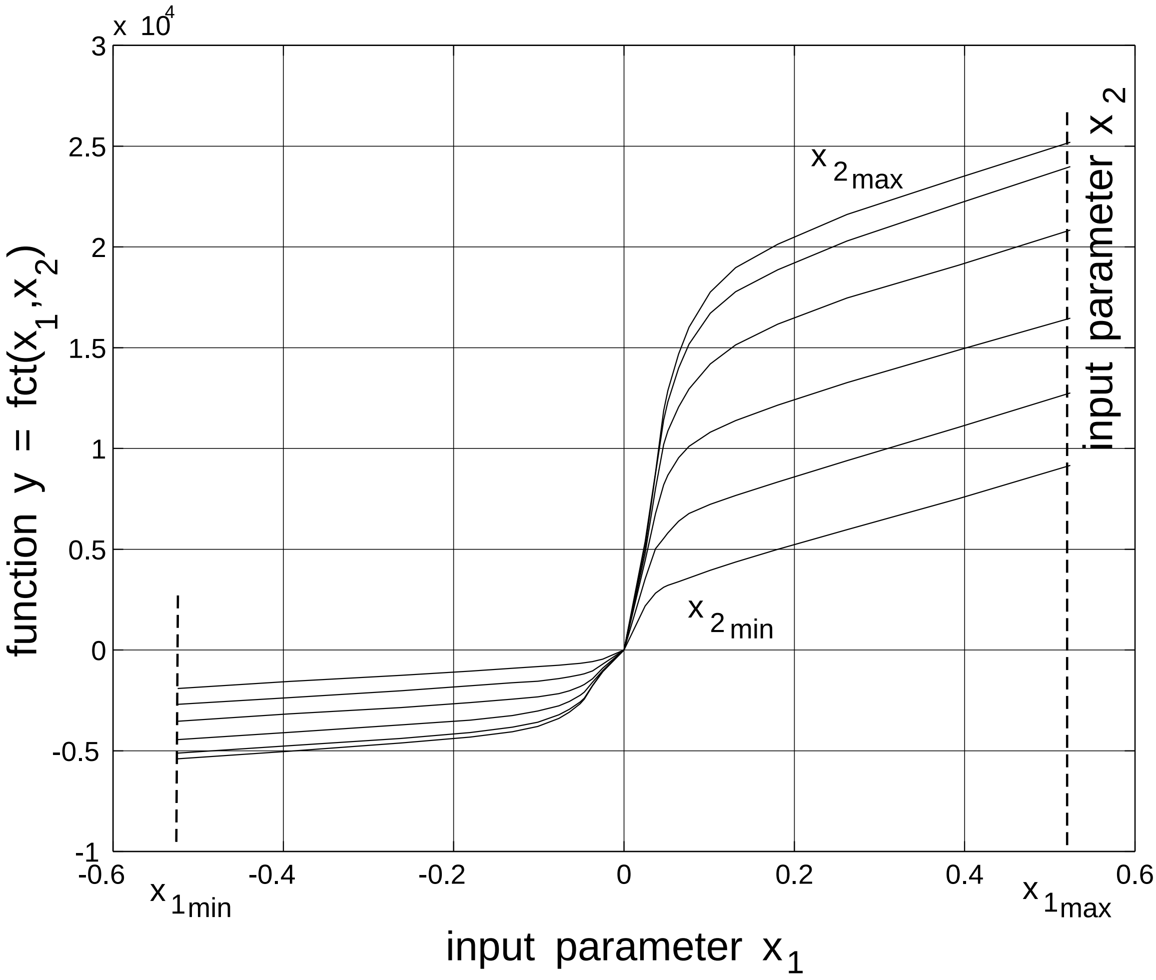

Figure B.3: 2D-Function y = fct(x1, x2)

- 3. The inverse evaluation x_2=fct(x_2,y) (force.par(22)=2): The function value $y = c_2\,u_2$ and the first input parameter $x_1 = c_1\,u_1$ are given. The fist input parameter x_2 is evaluated. Output $ov(1) = x_2 + force.par(13)$.

Presently only linear interpolation in the twodimensional parameter array is possible. A smooth transition from one segment to the next is defined by the smoothing ranges for both input parameters (force.par(24),force.par(25)) and, in case of inverse evaluation, for the function value (force.par(26)). This smoothing may result in a small deviation from the given table value. This can be bad in some cases. (For example a non-vanishing damper force for missing damper velocity). For this special case of vanishing input parameter the user can require the equality of interpolation and table value by force.par(27).

Appendix C

Landing gear model in SIMPACK

The model of landing gear used for this work is a generic landing gear of a regional aircraft. The rigid landing gear was prepared in SIMPACK as described in the Section 4.2. A multi-body model of a landing gear is prepared using various body elements which are connected to each other by means of joints. There may be force elements at the joints or between two body elements which are explained in Section 4.2.2. A rigid model, though sufficient for the dynamic load calculations, can not simulate the deformations that landing gear parts might undergo. To study these effects flexible landing gear was prepared in Nastran for modal analysis.

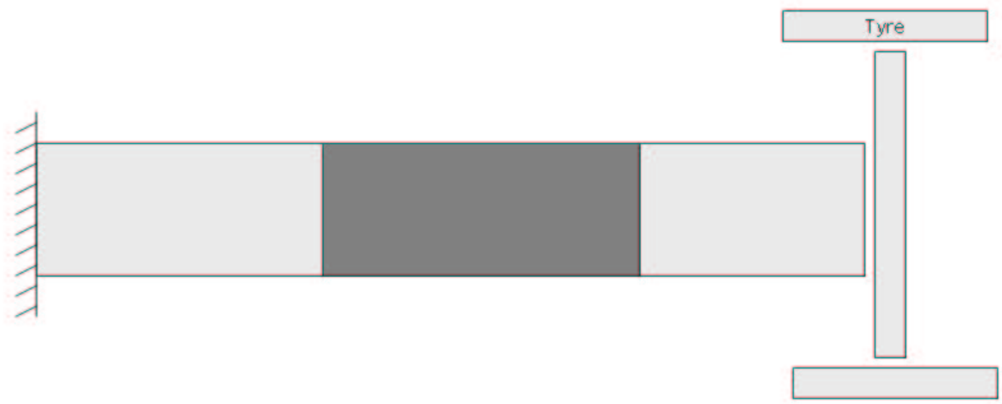

Figure C.1: Landing gear model in Nastran; constant stroke

The landing gear model is prepared in Nastran as a beam model with the help of data exchanged with the industry partner for a newly developed regional aircraft.

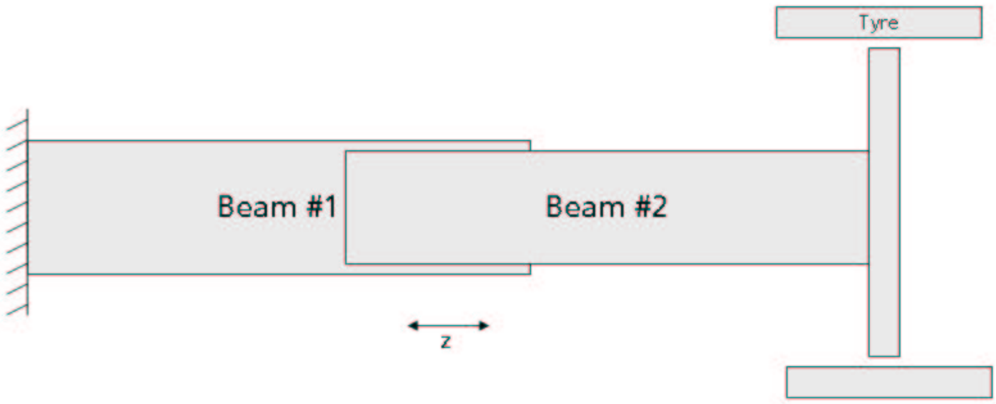

Figure C.2: Landing gear model in Nastran; variable stroke

The landing gear is modeled for different strokes and the results of the modal analysis are compared to the model and fine tuned to get similar eigen modes and eigen frequencies. To create the model in Nastran following data given in Table C.1 is used. The wheel axle is attached at the end of beam number two; tires are attached with the rotational degree of freedom around the y-axis. The wheels are represented by condensed masses. Two different models were prepared. Figure C.1 shows the model where it was modelled assuming a constant stroke. Landing gear is treated as three distinct parts with appropriate properties; dark grey portion shows the part of landing gear where shock-strut is inside the main fitting with a constant stroke. Figure C.2 shows the second model where each part of the landing gear was modelled separately in Nastran and imported into SIMPACK as explained earlier. In this case shock strut is having a z-translational degree of freedom.

Parameter	**Beam1**	**Beam2**
$D_{1,2}$	0.15 m	0.13 m
$L_{1,2}$	1.469 m	1.188 m
$t_{1,2}$	0.005 m	0.004 m
$E_{1,2}$	2.1E+11	2.1E+11
$\rho_{1,2}$	7895 Kg/m^3	7895 Kg/m^3

Table C.1: Data used for the Two Beam Landing Gear Model in Nastran.